論文の内容の要旨

論文題目　Stress Field Tomography Based on 3D Photoelasticity
（三次元光弾性に基づく応力場トモグラフィー）

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The objective of this thesis is to present a robust tomographic method for reconstructing 3D stress fields based on 3D photoelasticity. During last few decades, tensor field tomography has received more attention as it has important applications in various fields like medicine, engineering, physics, etc.. Stress field tomography based on 3D photoelasticity is considered one such important application in engineering and physics.　However, the inverse problem of 3D photoelasticity has not yet been solved in general case; to identify arbitrary state of stress without any limitation on identifiable state of stress and sensitivity of photoelastic material. A new method capable of reconstructing 3D distribution of general second order tensor fields called load incremental approach is proposed in this thesis for solving the inverse problem of 3D photoelasticity in general case. Based on this, a robust stress field tomographic method has been developed and it has been numerically and experimentally validated.

In 3D photoelasticity, a stressed object made of photoelastic material is observed from different directions using polarized light. While passing through the object the light vectors are changed according to the state of stress along light path and output light carry integrated information on state of stress as changes in phases and amplitudes of light vector. The inverse problem of 3D photoelasticity is to reconstruct the 3D distribution of stress tensor based on these light vector information.

The major difficulties in this inverse problem are its nonlinear and ill-posed natures. Both experimental and numerical techniques have been proposed to solve this nonlinear and ill-posed inverse problem. The experimental approaches degenerates the 3D problem into a series of 2D problems by obtaining photoelastic observations in high spatial resolution along light path, optically or mechanically slicing the object. These methods are either destructive or have practical problems. Instead of solving the original nonlinear and ill-posed inverse problem, most of the analytical approaches are based on a linear approximation valid only for weakly birefringent materials (i.e., less sensitive photoelastic materials). Less sensitive photoelastic materials have almost linear solution space in a considerable neighborhood of stress free state. Taking this advantage, most of the approaches to solve this inverse problem approximate the relation between output light vectors and stress components to be linear and thereby avoid the difficulties due to nonlinearity and ill-posedness. Since this linear approximation is valid only in limited neighborhood of the stress free state, all the methods based on this linear approximation have an upper limit on identifiable state of stress. Only the analytical methods based on this simplified solution space and experimental based treatments are currently used in practice. All the approaches to solve the general inverse problem of 3D photoelasticity are either limited to theory or numerical simulations.

We developed a load incremental approach for stress component identification as the first attempt to solve this inverse problem in general. As the name suggest, this method is based on a linear relation between increment of light vector and increment of stress components due to small changes in external load. This linear relation is obtained by considering the Taylor expansion of output light vector with respect to externally applied load and neglecting the higher order terms. Unlike the conventional methods, load incremental approach does not use this linear relation to simplify the solution space. Instead, load incremental approach uses the Newton-Raphson algorithm based on the linear relation between increment of light vector and increment of stress component to find the correct solution in the nonlinear solution space. However, the

Newton-Raphson algorithm cannot be directly used to find the solution for a given set of photoelastic observations since the solution space geometry is too complicated and the inverse problem is ill-posed. Load incremental approach adopts a suitable experimental procedure to treat the ill-posed nature. Starting from stress free state externally applied load on the object is increased in small steps and photoelastic observations are made at each step. Through this experimental procedure, the nonlinear solution space between output light and stress components is chopped into small segments so that each segment has a unique solution to the photoelastic observations made at that step. Since each these segments are still nonlinear, Newton-Raphson algorithm based on the linear relation between increment of output light and increment of stress components is used to identify the solution after each load increment. As long as the load increments given in the experiment are sufficiently small to chop the solution space in to small segments with a unique solution, the state of stress after each load increment can be successfully identified. State of stress up to arbitrary state can be identified in stepwise by increasing the external load in sufficiently small steps.

Since the equilibrium condition is not used, the solutions of load incremental approach for stress component identification are not necessarily stress fields. Therefore, this method can be considered as a 3D tomography of general second order tensor fields. However, this method is named as load incremental approach for stress component identification, as it is used for identifying distribution of stress components. This method is numerically validated and found that state of stress up to arbitrary state can be reconstructed as long as output light measurements are error free and load increments are sufficiently small. However, this method is highly sensitivity to unavoidable errors in photoelastic measurements like heat noise in CCD chips because each component have freedom to independently change according to the photoelastic observations.

With the aim of reducing the sensitivity to measurement errors in above explained stress component identification method, both the equilibrium condition and linear elasticity are introduced to the inverse problem of 3D photoelasticity. The introduction of equilibrium condition transforms the former 3D tomographic method for general second order tensor fields to 3D stress field tomographic method. This stress field tomographic method should be less sensitive to measurement errors since stress components are bounded together by equilibrium condition. When the equilibrium condition and linear elasticity are introduced, the only unknown is boundary conditions.

Therefore, this stress field tomographic method is named boundary condition identification.

In boundary condition identification, the unknown complicated boundary conditions are modeled with independent set of boundary condition modes and state of stress due to unit amplitude of each mode is found using a numerical method like FEM. The new inverse problem in boundary condition identification is to identify the amplitudes of boundary condition modes based on photoelastic measurements. It is shown that this inverse problem is also nonlinear and ill-posed. This nonlinear and ill-posed inverse problem is also solved using the load incremental approach. Through numerical and experimental validations, it has been shown that the load incremental approach for boundary condition identification is less sensitive to unavoidable errors in photoelastic measurements.

The major advantages of boundary condition identification are drastic reduction of number of unknowns and the robustness against measurement errors. Through modeling unknown boundary conditions as an independent set of modes, the number of unknowns in the original inverse problem is drastically reduced; millions of unknown stress components are reduced to few number of boundary condition amplitudes. As an immediate consequence, boundary condition identification needs fairly less computational power. A redundant set of equations can be obtained since the object can be observed using large number of light rays while the number of unknowns is a few. These redundant set of equations have different solution space geometries since different light rays passes through different portions of the object from different directions. Selecting only the equations with favorable solution space geometries for Newton-Raphson method, most of the numerical instabilities due to complicated solution space geometry can be avoided. The number of modes used in boundary condition identification has to be limited to the lowest 20 or lesser since the photoelastic observations do not carry accurate information on higher modes due to unavoidable measurement errors. As a result, boundary condition identification cannot identify the minor details of stress field corresponding to higher modes. However, this cannot be considered as a disadvantage of this method since this loss of minor details is due to experimental limitations. Some disadvantages of this method are necessity of prior knowledge on possible boundary conditions for selection of suitable set of modes and necessity of a numerical method to calculate the state of stress due to each boundary condition modes in models with huge degree of freedom.