論文内容の要旨

論文題目：

Pattern Formation in Reaction-Diffusion Systems with Advection

(流れ場のある反応拡散系のパターン形成)

小林　康明

In nonequilibrium steady state, many systems form what is called "dissipative structures", coined by Prigogine, which are macroscopic spatial structures sustained by the balance of energy injection and dissipation,. Extensive studies have been done in a variety of systems such as fluid convection and chemical reaction systems etc, all of which has the same class of basic patterns in common described by a small set of wavenumbers such as hexagon and stripe. Meanwhile, there has been a growing number of experimental reports on spatially localized structures in a wide range of systems.

In this dissertation we focus ourselves on the reaction-diffusion systems, multi-component concentration fields which are engaged in nonlinear reactions and diffuse in space following the Fickian diffusion equation. Reaction-diffusion systems are abstract chemical pattern-forming model which has a wide application to many real experimental systems. While a huge amount of work has been done on the reaction-diffusion systems, we give them a new insight by introducing advection terms. One of the motive is the fact that stable localized patterns have soliton-like identity and sometimes behave as particles, although they are different from solitons in several crucial aspects. Then it is quite natural to extend the interest to spatially extended patterns and consider them as a rigid structure, and the application of external flows on these patterns can be a good starting point on studying such property. Also, recently several models have been proposed for the experiments on the pattern formation of electroconvection of metallic powders, bacterial colonies, etc, which naturally entails advection terms in the expression of reaction-diffusion equations or similar types of equations. In this dissertation we also propose equations based on reaction-diffusion equations with self-generated advection term as a model of recent gas-discharge experiment.

This dissertation is organized as follows: In the first part we investigate a model reaction-diffusion system which is under an influence of a shear flow. The Brusselator model is adopted. First we investigate the amplitude equation of the Brusselator model near the onset of subcritical bifurcation for use as a reference state in studying the effect of shear . We obtain the upper-branch solution of hexagonal patterns which has a finite amplitude at the bifurcation point by utilizing a perturbation from the super-critical branch. Next, starting from this hexagonal solution we derive the nonlinear phase equation in the case of finite shear rate. In this equation the shear flow appears as an external body force acting on the elastic medium and hexagonal structures can be considered as a crystal. which is a further indication of the crystal-like behavior of the pattern. Also, as a further indication of crystal-like property, we find dislocation dynamics driven by the flow in hexagonal and stripe patterns. In special, we observed new dislocation dynamics which had not been seen in any chemical pattern-forming systems before, such as the propagation of the localized region which has a different texture from the hexagonal background, and the glide motion in the stripe background, which is usually observed in anisotropic systems like liquid crystal convection. We also investigate another chemical pattern-forming system called the Gray-Scott model and apply the linear shear on a hexagonally-ordered spots. Again we obtain the same sequence of transition from hexagon to stripe. Thus we expect that this sequence can be widely applied to many pattern-forming systems when the system is under an external flow and of course when the system intrinsically has the flow field, regardless of the mechanism of the pattern formation.

In the second part we construct a model equation describing intriguing localized patterns which has recently been observed in a gas-discharge experiment, where localized discharge regions form patterns. The model consists of a set of reaction-diffusion equations called the Gray-Scott model, augmented with a self-generated advection term. We numerically investigate this model to reproduce the experimentally obtained localized clusters and strings/loops, and the transition between the patterns. We further analyse the model by using a particle description and obtain the equation of motion for the center of mass of the localized structures. At this level the advection term appears as an attractive force between the particles. Existence of the steady state solution in the equation of motion is confirmed in the case of two-particle state, which implies the formation of steady localized clusters. The linear stability analysis is conducted to show that there is a critical strength of the flow field above which the distinct localized structures collapse into a connected structure, suggesting the formation of strings/loops. In addition to the success of reproducing the main feature of the experiment, this model is quite unique in that there is no such models which have closed loop solutions.

In both systems, we saw that the flow assumes an aspect of the force field from rigid patterns' point of view, regardless of the origin of the flow, intrinsic or extrinsic, and whether the pattern is spatially extended or localized.