論文内容の要旨

論文題目：Numerical studies on cosmological perturbations in braneworld
（和訳：ブレーンワールドにおける宇宙論的摂動の数値的研究）

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A theoretical framework to give a simple mechanism of the generation of cosmic structures is known as the inflationary theory. In this framework, the accelerated phase of the cosmic expansion took place and the spacetime inhomogeneity was generated quantum-mechanically in the course of the accelerated expansion, called the inflation. The quantum-generated fluctuations may transit to classical fluctuations, namely, the fluctuations of the spacetime curvature and the inflationary gravitational wave background (IGWB). The curvature fluctuations evolves and transferred to the density perturbations of baryons and photons filling the early universe. When the size of the universe becomes large enough to decouple the interactions (called decoupling), the photons started to travel freely through the universe, whose surface viewed from us is called Last Scattering Surface. We observe the photons as the Cosmic Microwave Background (CMB).

Evolution of such tiny fluctuations is treated theoretically by cosmological perturbations, in which we consider the spacetime fluctuation coupled with matter in the homogeneous and isotropic cosmology.

Observing primordial fluctuations generated during inflationary epoch may allow us to ask a fundamental question: “Why is our universe described as a four-dimensional spacetime ?” or “Is there a possibility that we live in a higher-dimensional spacetime ?”. This kind of questions emerges from the unified theory such as string theories. There is so far no experimental and observational evidence that our universe is such a higher-dimensional spacetime. With growing observational technologies, the day may be not far off that future observations stand our vision of the universe on its head. Hence it is very important and urgent to study the cosmological perturbations in the framework of higher-dimensional cosmology.

What makes the theoretical investigation possible is the appearance of the Randall–Sundrum (RS) single brane model (RSII model). In this model, we live on a brane, which is a three-dimensional hypersurface embedded in a five-dimensional anti-de Sitter bulk spacetime. In this thesis, we study the evolution of inflaton perturbations (scalar perturbations) and IGWB (tensor perturbations) in a cosmological setup based on the RSII model. In this setup, the brane is moving in the five-dimensional spacetime, and two high-energy effects play an important role in the evolutions of these perturbations in the high-energy regime of the universe. One of the effects comes from the high-energy correction to the Friedmann equation, leading to a peculiar cosmic expansion at the early stage of the universe; the other is due to the existence of the extra-dimensional (bulk) metric perturbations, which can be interpreted as the energy loss of the perturbations on the brane.

As for the IGWB, we focus on its evolution after the inflation and estimate an observed spectrum of the IGWB. We assume that our universe after the inflation is filled by a perfect fluid whose equation-of-state is described by $p = w\rho$. Using the spectral collocation method, we perform the numerical simulations in the cases with $w = 0, 1/3, 1$ which corresponds to the matter-dominated, the radiation-dominated, and the stiff-matter-dominated universe, respectively. At the initial time of each simulation, we set the IGWB whose wave-length is sufficiently larger than the Hubble horizon. The effect of bulk metric perturbations appears at the horizon crossing (re-entry). Then we find following things :

- The two effects cancel each other, resulting that the spectrum of IGWB becomes almost same one as predicted in the standard four-dimensional theory [see Fig. 1].
- This cancellation occurs only in the radiation dominated case ($w = 1/3$). In cases with $w = 0, 1$, the spectra becomes different from the four-dimensional ones.
- There is a relation between the Lorentz factor of the moving brane and the excitation of the Kaluza-Klein modes which are gravitational waves escaping from the brane into the bulk. This relation is insensitive to the parameter w. Using this universal relation, the spectrum of IGWB in the RSII model becomes

  $$\Omega_{\mathrm{GW}} \equiv \frac{1}{\rho_{\mathrm{c}}} \frac{d\rho_{\mathrm{GW}}}{d\log f} \;\propto\; f^{\frac{3w-1}{3w+2}} \;\text{ for } f \gg f_{\mathrm{crit}},$$

  where $f_{\mathrm{crit}} \approx 5.6 \times 10^{-5}$ Hz if the curvature scale of the anti-de Sitter spacetime is 0.1 mm, and $\rho_{\mathrm{c}} = 3H_0^2/8\pi G = 9.8 \times 10^{-30}$ g/cm^3 means the critical density of the universe and ρ_{GW} the energy density of the IGWB.

In the basis of the universal relation, we discuss the effect of the Lorentz factor of the moving brane using a simple toy model.

As for the inflaton perturbation, we investigate its evolution during the inflation with a specific brane inflation model proposed by R. M. Hawkins and J. E. Lidsey. The inflaton perturbation and the bulk metric perturbations are expressed by the Mukhanov-Sasaki variable, $Q(t)$, and the

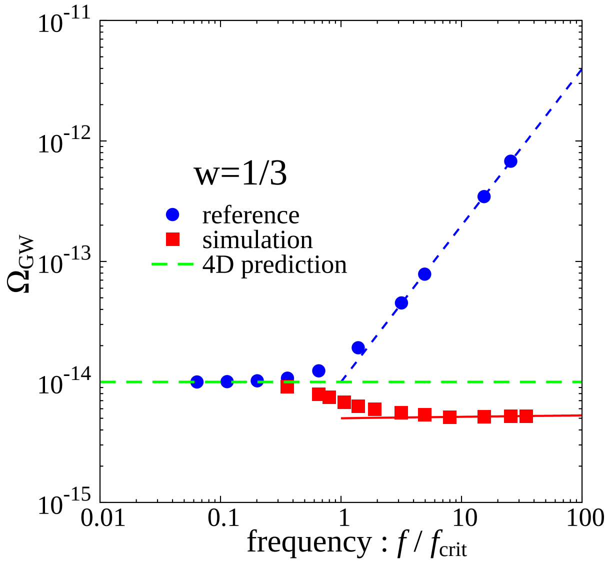

図 1: The energy spectrum of the IGWB around the critical frequency. The filled circles represent the spectrum caused by the non-standard cosmological expansion of the universe. Taking account of the KK-mode excitations, the spectrum becomes the one plotted as filled squares. Particularly, in the asymptotic region depicted in the solid line, the frequency dependence becomes almost same as the one predicted in the four-dimensional theory (long-dashed line).

master variable, $\Omega(t, y)$, respectively, where y denotes the bulk coordinate. These quantities are coupled with each other via the junction conditon imposed on the brane. In order to quantify the effect of the bulk metric perturbations, $\Omega(t, y)$, we adopt a simplified initial condition, in which no bulk metric perturbation is present at the initial time. By means of the spectral collocation method, we solve numerically the coupled equations of the metric perturbations and the inflaton perturbations. As a result, we found following things :

- The comoving curvature perturbation $\mathcal{R}_c \equiv -HQ/\dot{\phi}$ becomes constant on the super-horizon scales (at late time), as shown in Fig. 2, which is the same feature as one observed in the standard four-dimensional theory.
- Comparing with the curvature perturbations without considering the bulk metric perturbations, the amplitude of $\mathcal{R}_c$ in the present case is suppressed on the sub-horizon scales, as shown in Fig. 3.

The latter result leads to the impossibility of neglecting a coupling to the bulk gravity on small scales. Therefore, unlike the four-dimensional case, one cannot naively treat the initial inflaton field as a free massless field. This fact means that the observed spectra of the curvature perturbations or density perturbations become sensitive to the bulk metric perturbations at the initial time.

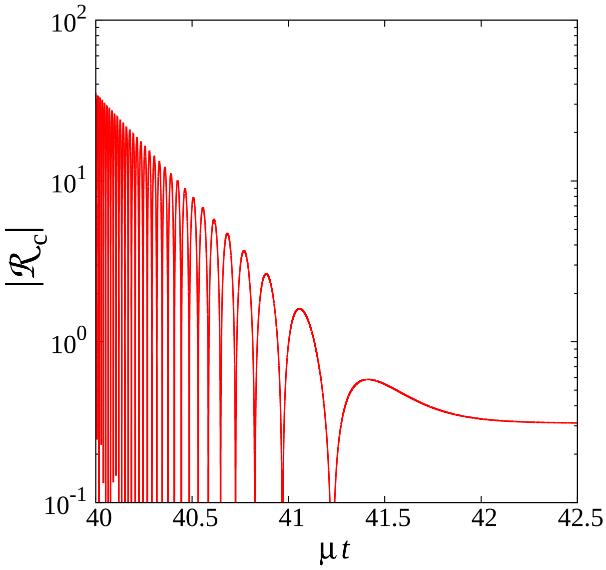

図 2: The evolution of curvature perturbations $\mathcal{R}_c$ in the inflationary epoch.

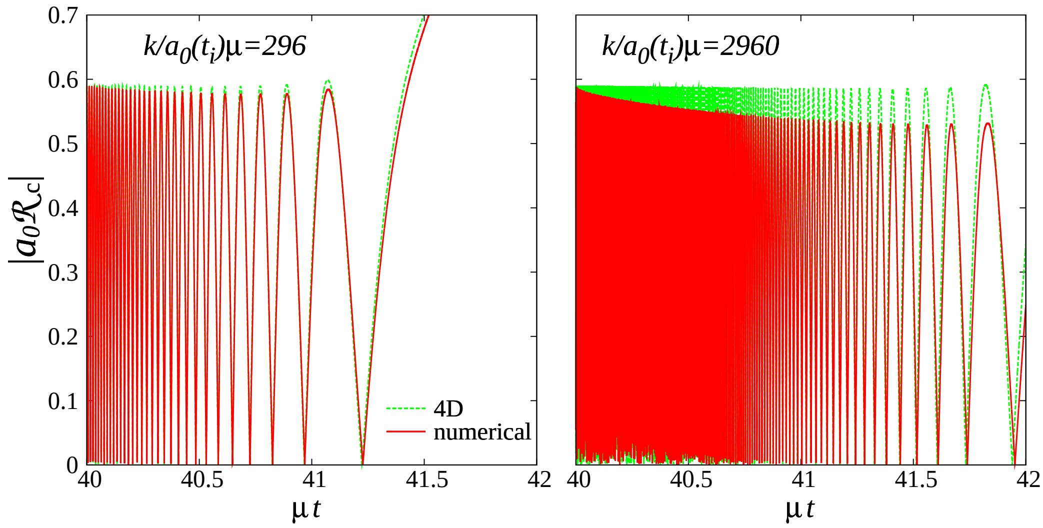

図 3: The curvature perturbations (multiplied by the scale factor) in the inflationary epoch for a long-wavelength mode (*left*) and for a short-wavelength mode (*right*). The solid lines represent numerical results and the dashed lines show the 4D predictions obtained by neglecting the coupling to the bulk metric perturbations.