

An analytical study for 1-dimensional steady flow in volcanic conduits: Origin of diversity of eruption styles

(一次元定常火道流の解析的研究：噴火タイプの多様性の成因)

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As silicic water-rich magma ascends to the surface and decompresses, volatiles exsolve. If exsolved gas remains trapped in the magma, volume fraction of gas increases and the magma inflates. As a result, magma fragmentation occurs and the flow changes from bubbly flow to gas-pyroclast flow, leading to explosive eruptions which form massive columns (Fig. 1). In contrast, if loss of gas from the magma occurs, inflation of the magma is suppressed and the magma does not fragment up to the vent, leading to effusive eruptions which form lava domes or lava flows (Fig. 1). Therefore loss of gas from the magma is a key process which causes diverse eruption styles of silicic magmas. In this paper, the effects of vertical relative motion between gas and liquid (vertical escape of gas) on diversity of eruption styles are investigated on the basis of a model for 1-dimensional steady flow in volcanic conduits. In our model, the vertical relative motion between the gas and the liquid is allowed and a transitional region ('permeable flow region') is introduced between the bubbly flow region and the gas-pyroclast flow region. In this region, both the gas and the liquid are continuous phases, allowing the efficient vertical escape of gas through the permeable structure.

The features of the conduit flow with the relative motion between the gas and the liquid are described by non-dimensional numbers:

$$\alpha \equiv \frac{8\mu q}{\rho_l^2 g r_c^2}, \quad \gamma \equiv \frac{\rho_l^2 g r_c^2}{8\mu q_{\max}} \quad \text{and} \quad \varepsilon \equiv \frac{8\mu k}{\mu_g r_c^2 (1 - \phi_{\text{crit}})},$$

where μ is the liquid viscosity, q is the magma flow rate, ρ_l is the liquid density, g is the acceleration due to gravity, r_c is the conduit radius, k is the magma permeability, μ_g is the gas viscosity, ϕ_{crit} is

the critical gas volume fraction for fragmentation, and q_{\max} is the maximum mass flow rate, which is expressed by $q_{\max} \equiv P_{f0}/\sqrt{n_0RT}$. Here P_{f0} is the pressure at the time of magma fragmentation for the flow without the relative motion, n_0 is the initial H₂O content, R is the gas constant, and T is the magma temperature. The parameter α represents the ratio of effects of wall friction and gravitational load, the parameter γ represents the effect of conduit conductivity r_c^2/μ , and the parameter ε is defined as the ratio of effects of liquid-wall friction force and liquid-gas interaction force in the permeable flow region, and represents the efficiency of gas escape from magma. When the relative motion is taken into account, the pressure at the time of magma fragmentation (P_f) is analytically expressed by

$$P_f \sim \frac{1}{\varepsilon + 1} \left\{ 1 - \frac{1}{1 + \frac{1 - n_0}{(1 - \phi_{\text{crit}})^2} \left(\frac{1}{\varepsilon} + 1 \right) \alpha} \right\} P_{f0}.$$

The magnitude of P_f decreases as the magma flow rate (α) decreases or the efficiency of gas escape (ε) increases, because the effect of gas escape suppresses the increase in the gas volume fraction accompanied by magma ascent. When α is so small or ε is so large that P_f is below the atmospheric pressure (P_a), the flow reaches the vent before fragmentation. On the other hand, when α is so large or ε is so small that P_f is greater than P_a , the flow reaches the vent after fragmentation. The steady solutions of conduit flow in which the flow reaches the vent before and after fragmentation correspond to effusive and explosive eruptions, respectively. The problem of the 1-dimensional steady conduit flow model is formulated as a problem to find a non-dimensional magma flow rate α as a function of the parameters related to magma properties and geological conditions (e.g., γ and ε) under given boundary conditions. A graphical method to systematically find α is proposed. In the graphical method, the curves which represent the variations of the length of the region before fragmentation (L_b) positive sign downwards and that of the region after fragmentation (L_g) as a function of α (L_b curve and L_g curve) are used (Fig. 2). The steady solution of conduit flow for a given total length of the conduit (L_{total}) is obtained by the relationship of $L_b(\alpha) + L_g(\alpha) = L_{\text{total}}$. The steady solution can be graphically obtained by finding the position where a vertical bar with the length of L_{total} contacts with L_b and L_g curves (Fig. 2). When the bar is located in the region where $L_g = 0$, the solution corresponds to effusive eruptions. On the other hand, when the bar is located in the region where $L_g > 0$, the solution corresponds to explosive eruptions (Fig. 2).

The numbers and the types of the steady solutions of conduit flow (i.e., the assemblage of the steady solutions) largely depend on parameters related to magma properties and geological conditions. On the basis of the graphical method, the relationship between the assemblage of the steady solutions and the magma properties or the geological conditions is investigated through the following four steps. In the first step, the characteristics of L_g curve are described using the critical values of ε (ε^*) and α (α_1 , α_2 and α_3), which are given by

$$\varepsilon^* \equiv \frac{P_{f0}}{P_a} - 1, \quad \alpha_1 \sim \frac{(1 - \phi_{\text{crit}})^2}{1 - n_0} \left(\frac{\varepsilon^*}{\varepsilon} - 1 \right)^{-1}, \quad \alpha_2 \sim \frac{1}{\gamma(\varepsilon^* + 1)} \quad \text{and} \quad \alpha_3 \sim \frac{1}{\gamma(\varepsilon + 1)}.$$

When ε is larger than ε^* , $L_g = 0$ independent of α . On the other hand, when $\varepsilon < \varepsilon^*$, the region where $L_g > 0$ exists in the range of $\alpha_1 < \alpha < \alpha_3$; $\alpha_1 < \alpha < \alpha_2$ corresponds to the regions where L_g increases with increasing α and L_g is independent of α , and $\alpha_2 < \alpha < \alpha_3$ corresponds to the region where L_g decreases with increasing α (Fig. 2). In the second step, the characteristics of $L_b + L_g$ are described using the critical values of $L_b + L_g$, L_1 , L_2 , L_3 , L_{\max} and L_p . Here L_1 , L_2 and L_3 are the values of $L_b + L_g$ at $\alpha = \alpha_1$, α_2 and α_3 , respectively, L_{\max} is the maximum value of $L_b + L_g$, and L_p is the value of $L_b + L_g$ in the limit of $\alpha \rightarrow 0$. These critical lengths are given by

$$L_1 \sim \frac{1}{1 + \alpha_1} \frac{P_0}{\rho_1 g}, \quad L_2 \sim \frac{1}{1 + \alpha_2} \frac{P_0}{\rho_1 g} - \frac{n_{f2} RT}{g} \ln \frac{P_a}{P_{f2}}, \quad L_3 \sim \frac{1}{1 + \alpha_3} \frac{P_0}{\rho_1 g},$$

$$L_{\max} \sim \frac{1}{1 + \alpha_1} \frac{P_0}{\rho_1 g} - \frac{n_{fm} RT}{g} \ln \frac{\varepsilon + 1}{\varepsilon^* + 1} \quad \text{and} \quad L_p \sim \frac{P_0}{\rho_1 g},$$

where P_0 is the pressure at the magma chamber, P_{f2} is the value of P_f at $\alpha = \alpha_2$, and n_{f2} and n_{fm} are the gas mass fractions at $P_f = P_{f2}$ and $P_{f0}/(\varepsilon + 1)$, respectively. In the third step, the relationships among the critical lengths (L_1 , L_2 , L_3 , L_{\max} and L_p), L_{total} and the assemblage of the steady solutions are systematically investigated. Finally, in the fourth step, those relationships are summarized using a simple regime map in the parameter space of ε , γ and L_{total} (Fig. 3). The regime map illustrates the complex relationship between the assemblage of the steady solutions and the magma properties or the geological conditions. According to the regime map, only a single solution of effusive eruption exists when ε is larger than ε^* . On the other hand, when $\varepsilon < \varepsilon^*$, the assemblage of the steady solutions is classified into the following five types: (1) Ef, (2) Ex, (3) Ef+Ex, (4) Ex+Ex, and (5) Ef+Ex+Ex, where Ef and Ex represent the solutions of effusive and explosive eruptions, respectively. Here (3), (4) and (5) represent that there exist the multiple steady solutions.

When the effect of liquid viscosity change during magma ascent is taken into account, we can define the values of ε in lower (ε_L) and upper (ε_H) regions of a conduit with low (μ_L) and high (μ_H) viscosities, respectively. According to our analytical result, when $\varepsilon_L < \varepsilon^* < \varepsilon_H$, the effusive solution continues to exist even if the pressure at the magma chamber changes substantially. In contrast, when $\varepsilon_L < \varepsilon_H < \varepsilon^*$, the transition between the effusive and explosive solutions is possible with the slight change in the pressure at the magma chamber. The observed eruption styles and the estimated magma properties and geological conditions for well-documented eruptions indicate that most values of ε_L and ε_H for the eruptions in which only a effusive eruption style was observed show that $\varepsilon_L < \varepsilon^* < \varepsilon_H$, whereas most values of ε_L and ε_H for the eruptions in which both effusive and explosive eruption styles were observed show that $\varepsilon_L < \varepsilon_H < \varepsilon^*$ (Fig. 4). A good consistency between the analytical result and the observations indicates that the complex transition between effusive and explosive eruption styles observed in nature can be explained on the basis of our analytical result.

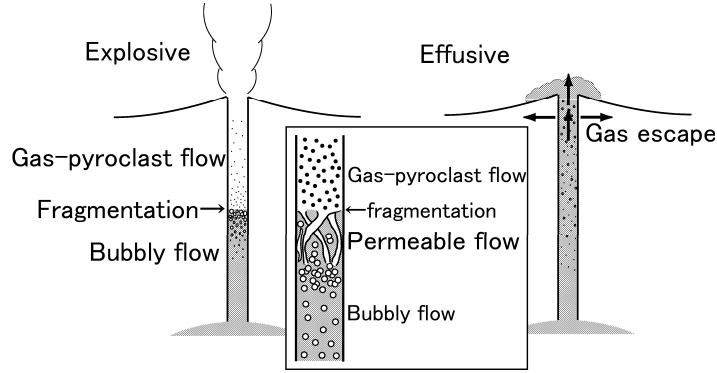


Fig.1. Mechanism for diverse eruption styles and conduit flow model in this paper.

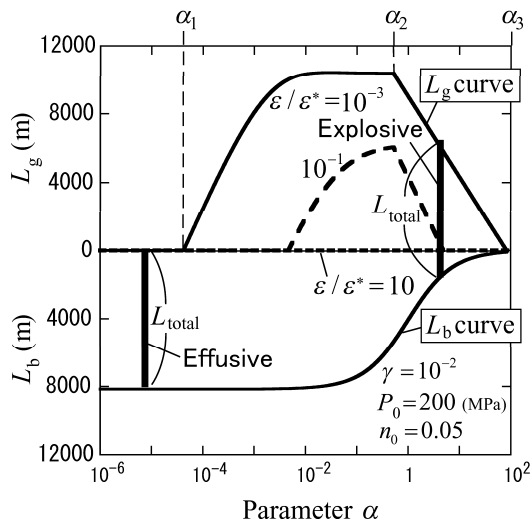


Fig.2. Graphical method to obtain steady solutions of conduit flow using L_b curve and L_g curve.

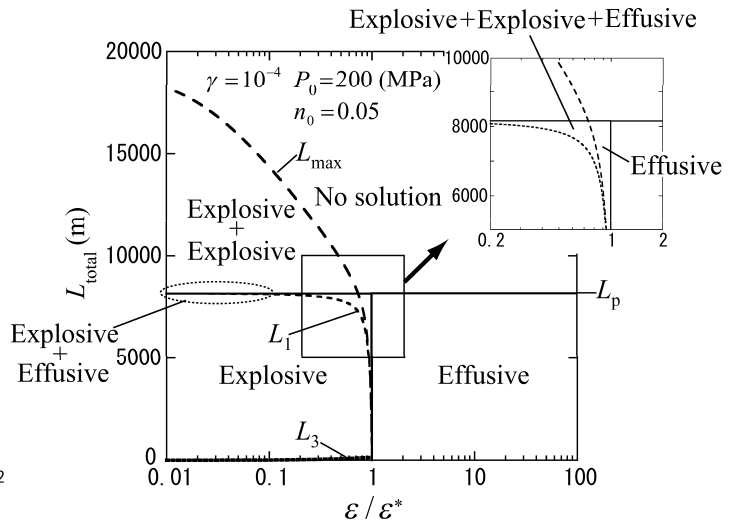


Fig.3. Regime map for assemblage of steady solutions of conduit flow.

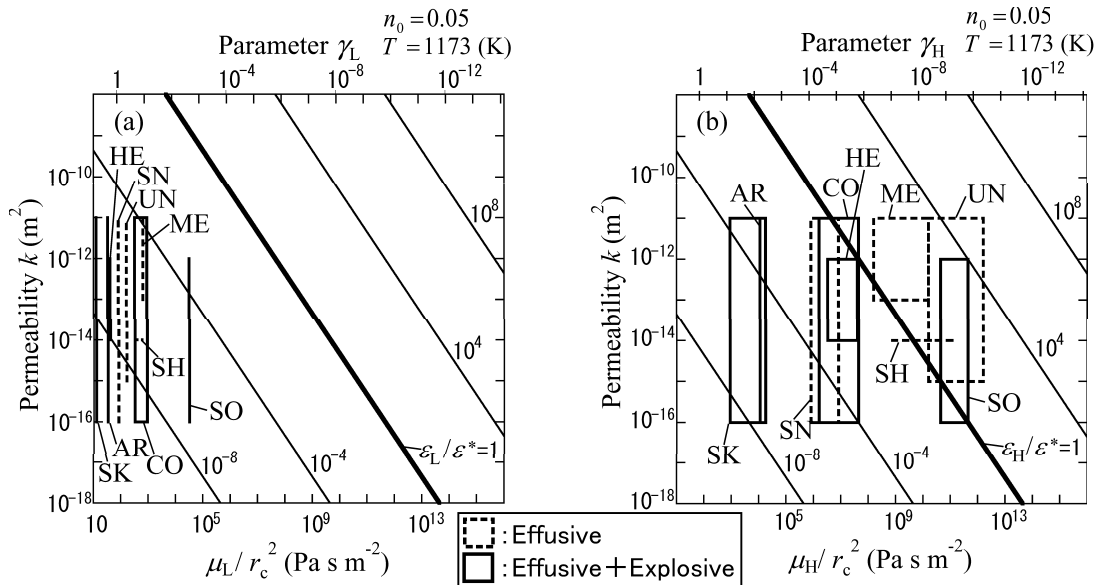


Fig.4. Comparison between analytical result and observations. (a) Relationships among k , μ_L/r_c^2 and ϵ_L/ϵ^* ; (b) relationships among k , μ_H/r_c^2 and ϵ_H/ϵ^* . UN-Unzen (1991-1995); ME-Merapi (1986-); SH-Shiveluch (2001-2004); SN-Santiaguito (1922-2002); HE-Mt. St. Helens (1980); SO-Soufriere Hills (1995-1999); CO-Colina (1988-); SK-Sakurajima (1914); AR-Arenal (1968-).