論文の内容の要旨

Studies on Modal Calculi すべて大文字
(様相計算に関する研究)

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The analogy between mathematical logic and typed λ-calculi has been formulated as formal correspondence between the theory of intuitionistic propositional logic and simply typed λ-calculus, known as Curry-Howard isomorphism. It maps formulas, proofs, and reductions of proofs in intuitionistic propositional logic into types, λ-terms, and reductions of λ-terms, respectively. As extensions are made to intuitionistic propositional logic to increase its expressibility, resulting in predicate logic, second-order logic, etc., simply typed λ-calculus has also been extended and corresponding λ-calculi have been formulated. In particular, the extension of simply typed λ-calculus corresponding to predicate logic is called λP.

In this thesis, according to Curry-Howard isomorphism, we construct an extension of simply typed λ-calculus, called strong $\lambda\Box$, which corresponds to an intuitionistic fragment of the minimal modal logic based on the notion of necessity. The extended λ-calculus can represent the so-called staged computation, where modalities correspond to different stages of computation.

Having various extensions of the theory of propositional logic and simply typed λ-calculus, it is of extreme interest to compare the expressive power of the extensions. In this thesis, we focus on correspondence between first-order predicate logic and modal logic. In mathematical logic, a translation from modal formulas into formulas in predicate logic is defined and the image of modal formulas by the translation is characterized by bisimulations. In this way, the notion of universality in predicate logic is shown to subsume the notion of necessity in modal logic.

On the analogy of this correspondence, we studied correspondence between proofs in first-order predicate logic and proofs in modal logic. In the thesis, we give a complete and full translation from strong $\lambda\Box$ into λP. A translation between two calculi is said to be complete when the translation preserves equations in the source calculus, and equations in the image of the translation can be pulled back to equations in the source calculus. Furthermore, it is called full if the translation does not add any junk. The existence of a complete and full translation means that the structure of the source calculus is preserved in the target calculus. Our result is about proofs which have not been dealt with in mathematical logic, i.e., the notion of universality is shown to subsume the notion of necessity more strongly. In this sense our result is a refinement of the above traditional result in mathematical logic.

In addition, by observing the image of other kinds of modalities in λP, such as intuitionistic T, K4, and S4, we obtain knowledge on those modalities in general.

We also investigated existing typed λ-calculi corresponding to modal logics, and focused on that there exist two kinds of context formulations for construction of such calculi. One is formulated by single contexts, e.g., λ^{SS4}, and the other is formulated by multi contexts, e.g., λ^{MS4}. We gave sound and complete translations between λ^{SS4} and λ^{MS4}, and claimed that the difference of context formulations raises no essential difference.