論文内容の要旨

Beilinson-Drinfeld chiral algebras for del Pezzo surfaces

(del Pezzo曲面に対するBeilinson-Drinfeldのカイラル代数)

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Recently the chiral algebra of Beilinson-Drinfeld draws much attention in the mathematical physics of superstring theory. Naively, this is a holomorphic conformal field theory with integer graded conformal dimension, whose target space not necessarily has the vanishing first Chern class. We usually exclude such a situation from our consideration since non-linear sigma model on such a target manifold contains a logarithmic divergence and hence a non-vanishing beta function. Scale invariance will be lost in such a situation and one can not apply the method of CFT (Conformal Field Theory). This algebra has two ways of definition: one is that of Malikov-Schechtman-Vaintrob called the chiral de Rham complex which glues affine patches, and the other is that of Kapranov-Vasserot by gluing the formal loop space. We will use the method of Malikov-Schechtman-Vaintrob in order to compute the gerbes of chiral differential operators.

If we use the method of chiral de Rham complex, when the manifold is covered by, say, 4 patches U_0, U_1, U_2, U_3, one first considers successive transformations $U_i \rightarrow U_{i+1}$. In the end one finds that under the total coordinate change $U_0 \rightarrow U_1 \rightarrow U_2 \rightarrow U_3 \rightarrow U_0$ the fields do not quite come back to their original values but there appears an additional term. Namely, there exists an obstruction or anomaly for a consistent CFT in such a system. It has been suggested that the obstruction is related to the first Pontryagin class of the manifold [Gorbounov-Malikov-Schechtman, Witten, Nekrasov, et.al.].

In this paper, we will explicitly confirm by 2 ways that the two independent ansatzes of Witten's (0,2) heterotic strings and Nekrasov's generalized complex geometry are consistent in the case of CP^2. Note that this target space has 3 affine patches and is expected to has the 1st Pontryagin anomaly. One way is by step by step careful OPE (Operator-Product-Expansion) calculation and the other is the computation of the anomaly 2-form -- the 2-cocycle of the chiral de Rham complex -- in terms of coordinate transformation Jacobian matrices. We also compute the anomaly 2-forms in the case of 2 dimensional toric Fano manifolds (toric del Pezzo surfaces) of all degrees, by blowing up the generic 1,2,3 points of CP^2. These coincide with the computation of the Hirzebruch-Riemann-Roch theorem. The most notable case is the 1 point blowup, where the total gauge invariant anomaly vanishes.

In chapter 2 of this thesis we start with the general theory of β, γ system

(conformal dimensions of γ, β are 0 and 1, respectively) where γ field is identified as the local coordinate of the manifold and β field is identified as a 1-form. Following [Malikov-Schechtman-Vaintrob] and Nekrasov we discuss the transformation laws of γ, β system under coordinate change so that their OPE is preserved. Then in chapter 3 we discuss the case of CP^2 as the target manifold and reproduce the result of Witten. In chapter 4 we consider the cases of del Pezzo surface, which are obtained by making blow-up at 1,2, and 3 points from CP^2. The case of 1 point blow-up has the vanishing Pontryagin anomaly. We compute the obstruction for these cases and find that they are proportional to the first Pontryagin class of the manifolds. Computations are somewhat lengthy but straightforward. Non-trivial aspect of the computation is the change in the convention of the normal ordering when one goes to a different coordinate patch and we have to make a careful analysis. In chapter 5 the siginficant future direction towards its application to the geometric Langlands program is also discussed. In Appendix A we explain the historical background of Wess-Zumino-Witten theory. This theory is used in the ansatz of Nekrasov as an antisymmetric μ-term. In Appendix B we include some brief illustrations of the toric diagrams and blowups.