Essays on Capital Asset Pricing

Abstract

Part 1: Valuing Variable Annuities
Part 2: A Structural Model with Long term and Short term debts
Part 3: Equity Returns, Value Premium and Credit Risk

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The essays are concerned with two topics of capital asset pricing: Part 1 is valuing variable annuities, and Part 2 and 3 investigate modeling credit risk.

In Part 1, I propose a framework to evaluate variable annuities. A variable annuity is a financial contract sold by an insurance company, under which the insurance company agrees to make periodic payments to the insured beginning at some future date. A variable annuity offers a range of investment options, typically mutual funds that invest in stocks, bonds, money market instruments, or some combination of the three. The value of the investment of the annuity owner will vary depending on the performance of the investment options he chooses.

A common feature of variable annuities is the death benefit. If the insured dies, a person he selects as a beneficiary will receive the greater of: (i) all the money in the account, or (ii) some guaranteed minimum. Some variable annuities offer a "stepped-up" death benefit, which enables the insurer to "lock in" the investment performance and prevent a later decline in the amount that he expects to leave to his heirs. Other variable annuities offer a guaranteed minimum income benefit, which guarantees a particular minimum level of annuity payments regardless of investment losses he may incur. These features naturally carry charges, which will reduce the investment account value.

I show that the invested capital to a variable annuity can be decomposed into: (i) the reserve money in the account, (ii) options, (iii) fees paid to the mutual fund companies, and (iv) margin accruing to the insurance company. The first two components comprise value to the insured, and the last two accrue to the supply side companies. This view

provides a convenient method to double-check the computation of various components of value.

I also show that death benefit option, in the simplest and the most popular products, is a portfolio of European put options of differing maturities. Assuming that investment value follows a geometric Brownian motion, this component can be valued applying the Black-Scholes formula for European put options and using the "death rates" published by the Ministry of Health, Labour and Welfare. Stepped-up death benefit is a form of look-back options. An analytic solution is known for the case of continuous-time step-up. Since the actual products assume discrete-time (typically, quarterly) step-up, I value them using a trinomial lattice. Options on the income benefit are valued using the Black-Scholes formula.

I evaluate some typical products assuming a person purchases them at age 40 and at age 50. At the same time, I examine how various value components would change in response to the volatilities of the investment products, the length of the contract and so on. I also propose how the insurance fee rate should be economically determined and compare my solution to the actual rates.

In Part 2, I show a new framework of credit risk valuation by structural approach.

Structural approach of credit risk valuation computes stock and bond prices or likelihood of bankruptcy given the stochastic process of asset value. The first model of Merton (1974) showed that stock could be considered as a call option on the firm with the strike price equal to the face value of a single payment debt issue.

As for the Merton model, since the debt a firms holds is simplified with one kind, there is a problem that the short term likelihood of bankruptcy is incalculable even if a firm discloses the amount of short term and long term debt separately. If a firm has larger amount of short term debts, the short term default risk will be higher, while most of the existing models cannot introduce several debts separately into the model. As a result, such a model cannot apply to the pricing of credit derivatives such as credit default swap or a convertible bond. My first goal is to construct a model in which short term and long term debts reflect the term structure of default probability.

There is a paper in which models can input short term and long term debt separately: Geske (1977) extended Merton's simple model of one pure discount bond and only one option to default by allowing for two options to default on short term debt. In his model, the refinancing at the redemption of short term debt is done all by selling new stocks. In such case, since the stock after the maturity of the short term debt is an option on the asset's of the firm, the stock before the one is an option on an option, or a compound

option .

Although this model can input two kinds of different debentures, the problem is that, since short term default probability is affected not only by the amount of short term debt but by the amount of long term debt, the short term default probability almost does not depend on the ratio of amount of short term and long term debts, which is not realistic.

In Geske model, why do long term debts affect the short term probability? This is because the refinancing is assumed to be by selling new stocks. With this assumption, even if, at the maturity of short term debt, the asset value is larger than the face value of the short term debt, the firm does not always survive: only when the aseet value is large enough for stockholders to buy some new stocks, the firm does not bankrupt. Since the stock value depends on the amount of long term debt essentially, the short term default probability is also relevant to the amount of long term debt. In this part, I assume that refinancing is done by selling both a new subordinate bond with the same face value and new stocks. By this assumption, since the short term bond is redeemed mainly by selling the new subordinate debt, whose value is almost irrelevant to the junior long term debt, short term default probability depends less on the amount of long term debt.

Another goal is to examine how the credit risk is affected if the bondholder can postpone the redemption of a bond. In reality, in case the stockholders cannot redeem the face value of debt, the bondholders postpone the redemption of the bond. Using the model I built, I show that if the bondholder of the short term bond so behaves that he maximizes his own securities, it can be optimal for the bondholder of short term debts to postpone the redemption. This fact implies that the decision of the postponement depends on what else the bondholders of short term debt have at the same time. In this part I show how much the default probability of short period and long period changes after introducing the postponement of the redemption of short term debt into the model, or how much it depends on the governance of the firm.

One of the characteristic points of this model is that, since the face value of short tern bond is constant in the future in this model, the default risk does not depend on the path of the asset price, which makes the model clear and applicable to credit derivative pricing. As an example of that, I price a convertible bond.

In Part 3, I investigate the relationship between credit risk and equity premium.

As Part 2, in years, most of the analyses on credit risk have focused on default probability or the pricing of defaultable securities such as bonds or credit derivatives.

The relationship between credit risk and stock return is rarely thoroughly investigated.

Recently, however, researchers have investigated how the credit risk is reflected in the cross-sectional stock return, and most of the papers show results opposite to my intuition: firms with higher credit risk have much smaller average stock returns. Furthermore, this phenomenon appears even after the market, value and size effect are adjusted.

I investigate how credit risk and expected stock return are determined by economic primitives, such as tastes and technology, in the neoclassical framework with rational expectations. I show that, contrary to the conventional wisdom, a firm with higher credit risk can have less risky stock than the one with lower credit risk.

In the model I consider a firm with a zero-coupon bond. When the firm faces high default risk, the value of the cash flow after the maturity for stockholders is small. However, the firm pays dividend before the maturity to the stockholders, which is the main determinant of the stock value. Since the value of the cash flow before the maturity is more stable than the value of the cash flow after the maturity, the stock has low risk and low return, which explains the anomaly of the relationship between credit risk and stock return.

Garlappi and Yan(2006) is another theoretical paper based on the neoclassical framework explaining negative relationship between stock returns and credit risk. It explicitly introduces financial leverage in a simple equity valuation model and consider the likelihood of a firm defaulting on its debt obligations as well as potential deviations from the absolute priority rule (APR) upon the resolution of financial distress. In my model, even if investors expect no APR violation, the stock return can be lower as default probability is lower.

This part is related to two strands of the financial economics literature. On one hand it shares with a series of recent papers such as Berk, Green and Naik (1999), Gomes, Kogan and Zhang (2003), Carlson, Fisher and Giammarino (2004), Zhang (2005) and Obreja (2006), with the objective of explaining both the predictable variations in equity returns through time and the cross-sectional relation between equity returns and characteristics. However, these models focus on all-equity firms only except for Obreja (2006), while my model focuses on financially leveraged firms.

My theoretical model also shares with the credit risk literature on default probability and pricing of bonds. Important contribution to this literature include: Merton (1974), Black and Cox (1976), Geske (1977), Leland (1994), Leland and Toft(1996). From this perspective, the novelty in my model stems from the fact that the default probability is related to time-varying price of risk.