論文内容の要旨

Aspects of Integrability in AdS/CFT Duality

(AdS/CFT 双対性における可積分性の諸相)

岡村 圭祐

It is an established notion that the Standard Model successfully describes three out of the four types of fundamental interaction in nature. Those three unified are the electric, strong and weak force. Quantum field theories for the rest one to be unified, the gravity, has not been successful to the date due to the non-renormalisability. However, there is one very attractive candidate that can unify all the four natural forces. It is the string theory, which offers a consistent quantum field theory of gravity.

Remarkably, it is conjectured that string theories in certain backgrounds are dual to particular Yang-Mills theories. The best studied example of the duality is the one between the type IIB string theory on $AdS_5 \times S^5$ and the four-dimensional $SU(N)$ $\mathcal{N} = 4$ super Yang-Mills (SYM) theory, which is known as the AdS/CFT duality. It is generally hard to compare both theories in a direct manner, since the perturbative regime of one of the theories corresponds to non-perturbative regime of the other, namely, it is a duality of strong/weak type. This strong/weak nature is blessing and misfortune at the same instance. It can be seen as a blessing since once the duality is established, it enables us to study the non-perturbative regime of one of the theories by using the perturbative result of the other theory, and vice versa. It certainly leads to deep insights into non-perturbative formulation of large-N gauge theory and also string theory. It is a misfortune since it is hard to prove the duality itself. Definitely, we wish to turn this misfortune into the blessing.

Our strategy to test the AdS/CFT conjecture is to compare the spectrum of conformal dimensions of the gauge theory operators and that of the energies of string states, since the AdS/CFT predicts the exact matching of the two spectra. The full spectrum seems to depend on the 't Hooft coupling in a rather complicated way. Nonetheless, we have been able to compare the AdS- and CFT- spectra in a very sophisticated way, by virtue of a special hidden symmetry both theories are expected to possess, which enables us to solve the system almost exactly. The symmetry is called integrability.

The integrability arises in the string theory side as a classical symmetry of string worldsheet theory, while in the gauge theory side as a quantum symmetry of local operator mixing. The relation between the two kinds of integrability is not clear at first glance. With the aim of unifying them, much effort has been focused on constructing the Bethe ansatz equation which is valid for all regions of the coupling. Due to the Bethe ansatz approach, there has been considerable progress in matching the spectra recently. In this dissertation, we discuss how our understanding of the AdS/CFT duality has been improved by the integrability-based approaches. Below we describe the outline of the thesis, and sketch how the original works [1–5] are distributed along the thesis.

1. We begin with introduction to the AdS/CFT correspondence and some reviews and overviews of earlier results in Part I. In Chapter 1, we shall give a heuristic derivation for the Maldacena's original argument for the AdS/CFT. Then in Chapter 2, we briefly explain how integrability of the gauge and the string theory can be used to overcome the strong/weak difficulty.

2. In Part II, we discuss the integrability in gauge theory. In Chapter 3, after reviewing some relevant aspects of $\mathcal{N} = 4$ SYM, we discuss the one-loop renormalisation problem for the $SO(6)$ sector, and demonstrate how the resulting dilatation operator is identified with an integrable spin-chain Hamiltonian. Chapter 4 includes some introductory material to the Bethe ansatz method. In particular, we discuss the $SU(2)$ case in detail, for which we also review the higher-loop integrability.

3. In Part III, we move on to the $AdS_5 \times S^5$ string sigma model and see its classical integrability. We first review classical strings on $\mathbb{R} \times S^3$ carrying large spins in Chapter 5, then compare the energies with those of "long" SYM spin-chain states. For particular set of solutions, we explicitly see the mismatch of the energy coefficients of gauge and string theory at the third loop order, and explain the need of the dressing phase. We also review the finite-gap problem approach to the classical string spectrum. Furthermore, we present the most general elliptic string solutions on $\mathbb{R} \times S^3$ in Chapter 6, which is based on the original works [4, 6]. They are also interpreted as finite-gap solutions.

4. Having discussed the integrability observed in each end of the AdS/CFT correspondence, in Part IV, we try to unify them in the form of Bethe ansatz equation. Chapter 7 is based on the original work [3], in which we discuss the asymptotic spectrum of the $\mathcal{N} = 4$ SYM spin-chain. We show that the boundstates of Q magnons form a certain short representation of dimension $16Q^2$. We also derive the exact dispersion relation for the magnon boundstates by purely group theoretic means. In Chapter 8, we summarise the current knowledge about the "AdS/CFT S-matrix" that is supposed to interpolate between gauge and string theory S-matrix. Chapters 9 and 10 are based on the original works [1] and [2], respectively. On the string theory side, we construct string solutions corresponding to special worldsheet solitons, called dyonic giant magnons. The energy-spin relation for our solution is shown to precisely agree with the dispersion relation for the SYM magnon boundstates [1]. The scattering phase-shift computed directly from the dyonic giant magnon scattering also agrees with the one obtained using the conjectured S-matrix [2], thus giving a positive support for the conjecture. In Chapter 11, following the original work [5], we examine the singular structures of the conjectured S-matrix. By considering physical processes involving one or more on-shell intermediate particles belonging to the known BPS spectrum of [1], we perform further analyticity tests for the conjectured S-matrix.

5. Finally, Part V is devoted to the conclusion.

References

[1] H.-Y. Chen, N. Dorey, and K. Okamura, "Dyonic giant magnons," *JHEP* **09** (2006) 024, `hep-th/0605155`.

[2] H.-Y. Chen, N. Dorey, and K. Okamura, "On the scattering of magnon boundstates," *JHEP* **11** (2006) 035, `hep-th/0608047`.

[3] H.-Y. Chen, N. Dorey, and K. Okamura, "The asymptotic spectrum of the $\mathcal{N} = 4$ super Yang-Mills spin chain," *JHEP* **03** (2007) 005, `hep-th/0610295`.

[4] H. Hayashi, K. Okamura, R. Suzuki, and B. Vicedo, "Large Winding Sector of AdS/CFT," *JHEP* **11** (2007) 033, `arXiv:0709.4033 [hep-th]`.

[5] N. Dorey and K. Okamura, "Singularities of the magnon boundstate S-matrix." (to appear).

[6] K. Okamura and R. Suzuki, "A perspective on classical strings from complex sine-Gordon solitons," *Phys. Rev.* **D75** (2007) 046001, `hep-th/0609026`.

Map of the thesis: