論 文 内 容 の 要 旨

論文題目 Ultradiscrete soliton systems and combinatorial representation theory
(超離散ソリトン系と組み合わせ的表現論)

氏 名 坂 本 玲 峰

Consider the solvable lattice model described by quantum R matrix of $U_q(\widehat{\mathfrak{sl}}_n)$. Here we are considering models on the square lattice. By taking the limit $q \longrightarrow 0$, it is known that the model bears combinatorial structure. Such system is described by Kashiwara's crystal bases theory, and $q = 0$ analogue of the R matrix is called the combinatorial R. Using the combinatorial R, we can define a commuting family of the row-to-row transfer matrix $\{T_l\}_{l \in \mathbb{Z}_{>0}}$ by the usual prescription. If we ignore all horizontal lines of the model and look at vertical lines, then we obtain the so-called box-ball model, which was introduced by D. Takahashi and J. Satsuma in 1990. In this setting, operators $\{T_l\}_{l \in \mathbb{Z}_{>0}}$ yield the time evolution operators of the box-ball model.

Now we invoke the combinatorial bijection called the KKR bijection discovered by S. V. Kerov, A. N. Kirillov and N. Reshetikhin in 1986 as a combinatorial analogue of the Bethe ansatz. Here we are considering crystals associated to symmetric tensor representations. By the KKR bijection, we obtain complete sets of the action variables (i.e., constant of motion) and the angle variables (i.e. parameters which evolve linearly with respect to time) of the box-ball system. These action angle variables are called the rigged configurations. This result rely on highly nontrivial arguments and it demonstrates that the KKR bijection gives direct/inverse scattering transform of the box-ball system.

The original definition of the KKR bijection uses purely combinatorial language, and its representation theoretic origin is sought for a long time. We reformulate the KKR bijection in terms of the affine crystal bases. As an application of this formalism, we derive explicit analytic formula for the KKR bijection as the ultradiscrete limit of the tau functions for the KP hierarchy. This formula yields the general solution to the box-ball system as the byproduct. The result contains the general solution for the periodic box-ball system whose tau function is shown to be the ultradiscrete limit of the classical Riemann theta function.

As another direction of research, we consider tensor products of arbitrary Kirillov–Reshetikhin (KR) crystals. By Kirillov–Schilling–Shimozono (KSS) bijection, elements of tensor products of KR crystals have one-to-one correspondences with generalization of the rigged configurations. This is a natural generalization of the KKR bijection including the original one as special case. We give algebraic procedure to obtain the rigged configuration from tensor products of the KR crystals by using time evolution operators. This gives characterization of the KSS bijection as intrinsic property of tensor products of crystals.

These observations can be viewed as giving the first concrete example for the connection between the quantum inverse scattering formalism and the classical inverse scattering formalism.