論文の内容の要旨

論文題目　Sign-Solvability in Mathematical Programming
(数理計画法における符号可解性)

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Mathematical programming is a branch of mathematics concerned with optimization problems, which can be applied to a variety of engineering fields. By recent progress of computer technology and algorithms, we have come to be able to utilize a large class of efficiently solvable mathematical programming models. However, it is often difficult to develop a mathematical programming model which represents a practical situation exactly. This is because the input data of a model are subject to many sources of uncertainty including errors of measurement and absence of information. On the other hand, structural properties of a model are independent of such uncertainty. This motivates us to provide a combinatorial method that exploits structural information before using numerical information.

In this thesis, we focus on the sign pattern of a mathematical programming model, that is, the combinatorial arrangement of positive and negative numbers of the input data. The use of sign patterns for matrix analysis is called qualitative matrix theory, which originated in economics. The main aim of this thesis is to present a connection between qualitative matrix theory and mathematical programming. We are concerned with the following three problems related to mathematical programming.

The first one is to compute the inertia of symmetric matrices. The inertia of a symmetric matrix indicates the number of positive/negative eigenvalues. Quadratic forms are classified by the inertia of the coefficient matrices. A symmetric matrix is said to be sign-nonsingular if every symmetric matrix with the same sign pattern as the matrix is nonsingular. Hall, Li, and Wang showed that the inertia of a sign-nonsingular symmetric matrix is determined uniquely by its sign pattern. We present an efficient algorithm for computing the inertia of such matrices. The algorithm runs in $O(n\gamma)$ time for a symmetric matrix of order n with γ nonzero entries.

Secondly, we discuss solving linear programs from sign patterns. A linear program is said to be sign-solvable if the set of sign patterns of the optimal solutions is uniquely determined by the signs of the given coefficients. It turns out to be NP-hard to decide whether a linear program is sign-solvable or not. We then introduce a class of sign-solvable linear programs in terms of totally sign-nonsingular matrices, which can be recognized in polynomial time. For a linear program in this class, we devise an efficient combinatorial algorithm to obtain the sign pattern of an optimal solution from the given sign pattern. The algorithm runs in $O(m\gamma)$ time, where m is the number of constraints, and γ is the number of nonzero coefficients.

Finally, we examine sign-solvability of linear complementarity problems (LCPs). An LCP is said to be sign-solvable if the set of sign patterns of the solutions is uniquely determined by the signs of the given coefficients. We provide a characterization for sign-solvable LCPs whose coefficient matrix has nonzero diagonal entries. This characterization, which can be tested in polynomial time, leads to an efficient combinatorial algorithm to find the sign pattern of a solution for these LCPs. The algorithm runs in $O(\gamma)$ time, where γ is the number of nonzero coefficients.

These are the first attempts of qualitative analysis in mathematical programming which are independent of the magnitudes of the input values. In particular, the latter two results provide efficient algorithms to find the sign pattern of a solution. In both cases, the obtained sign pattern easily derives a solution itself by Gaussian elimination. Thus our results reveal new classes of mathematical programming problems which can be solved in strongly polynomial time.