論文の内容の要旨

論文題目　Polyhedral Realizations of Finite Distance Spaces and Applications to Directed Multiflow Problems
（有限距離空間の多面体的実現と有向多品種流問題への応用）

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The main aim of this thesis is to introduce a polyhedral realization, called the tight span, of a finite asymmetric distance space and its applications. A finite asymmetric distance space is a pair of a finite set and an asymmetric distance on the set. A distance does not necessarily satisfy the triangle inequality. The term "asymmetric" is used to emphasize that the symmetry condition is not imposed; namely, a distance between two elements depends on the direction to measure the distance. Hence, a symmetric distance is an asymmetric distance. Isbell (1964), Dress (1984), and Chrobak and Larmore (1994) independently established that for any metric space there exists an essentially unique maximal tight extension, which is called the injective envelope by Isbell, the convex hull by Chrobak and Larmore, and the tight span by Dress, respectively. Recently, Hirai (2006) has extended the notion of tight spans to symmetric distance spaces. In this thesis, we introduce the tight spans for asymmetric distance spaces.

We explain the concept of realizations of finite distance spaces through the best known realization: the tree representation of a tree metric. A tree metric is representable by the lengths of the paths between vertices of a certain tree with nonnegative edge lengths. In this case, the tree is called a tree representation of the tree metric. The tree realization is endowed with some desirable properties, such as uniqueness, tightness, and universality. It is known that a tree metric has a unique, up to isomorphism, tree representation. The tightness of a tree representation means here that every point on the tree, which might be a point on an edge, is contained in the geodesic between a pair of certain leaves. The universality of a tree representation can be explained as follows.

Given a tree metric on a set, we take points on the tree representation of the tree metric and add the points to the given set. Then, we obtain a new tree metric on the enlarged set. Moreover, the tree representation of the new tree metric is essentially the same as that of the originally given tree metric, that is, all the tree metrics that have a certain tree as their tree representations are already embedded into the tree.

The tight span proposed in this thesis is also provided with those properties. Actually, Dress introduced the tight span as a generalization of the tree representation of a tree metric. The tight span for a tree metric is a tree as a union of line segments and thus the tight span is the tree representation. Tight spans allow us to geometrically investigate the structure of a distance space.

Contributors in the theory of tight spans have also established applications of the theory, such as phylogenetic tree construction problems and multiflow or multi-commodity flow problems. This thesis includes investigations on those problems.

Buneman's and Bandelt and Dress' split decompositions of metrics are two well-known tools for phylogenetic tree construction problems. Bandelt and Dress' split decomposition of a metric induces a decomposition of the tight span for the metric. Hirai (2006) derives Bandelt and Dress' split decomposition as a special case of the polyhedral split decomposition of polyhedral convex functions. Our result shows that Buneman's split decomposition can be also understood as a special case of the polyhedral split decomposition. This result provides a geometric and unified interpretation of the two split decompositions.

In the multiflow theory, clarifying the bounded fractionality of a polyhedron of solutions is a central issue. We give a complete characterization of the class of the weighted maximum multiflow problems whose dual polyhedra have bounded fractionality. A key ingredient is the tight spansfor asymmetric distance spaces. The theory of tight spans provides a unified duality framework to the weighted maximum multiflow problems,and gives a geometric interpretation of combinatorial dual solutions of several known min-max theorems, such as Ford and Fulkersons' max-flow min-cut theorem and Frank's directed free multiflow theorem, in the multiflow theory.