論文の内容と要旨

論文題目　Combinatorial Algorithms for Continuous Optimization with Submodular Structure
（劣モジュラ構造を有する連続最適化問題の組合せ的アルゴリズム）

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A submodular function is a discrete analogue of a convex function. It enables us to describe a large class of practical problems and arises in substantial fields, including graph theory, economics and information theory. Since 1970s, research on submodular functions has revealed fundamental structures of efficiently solvable combinatorial optimization problems, and that area is still growing. One of the most exciting developments of recent years is a breakthrough in combinatorial algorithms for submodular function minimization (SFM). In this dissertation, we consider several problems with submodular structures, and design efficient algorithms with the aid of classical methods and state-of-the-art techniques.

First, we deal with a line search problem in a polyhedron associated with a submodular function. For this fundamental problem, we develop the first strongly polynomial algorithm by combining a fully combinatorial algorithm for SFM of Iwata and the parametric search method of Megiddo.

Next, we show that the recent SFM algorithm of Orlin can be embedded within a parametric minimization framework successfully, which results in a faster algorithm for parametric submodular function minimization. We also give a new application to the measurement of robustness of a submodular system.

Subsequently, we examine convex optimization problems over submodular constraints, and clarify the connections between some apparently different problems. In addition, we develop an efficient and general method for convex optimization over submodular constraints using parametric SFM algorithms.

Lastly, we treat convex relaxations for combinatorial optimization problems with submodular penalties. We propose the use of simple and recent algorithms for non-smooth convex optimization due to Nesterov. Our algorithms can be used to design fast and simple approximation algorithms for the original problems.