## 論文の内容の要旨

# 2D CURVE AND 3D SURFACE REPRESENTATION USING IMPLICIT POLYNOMIAL AND ITS APPLICATIONS (陰関数を用いた 2D 曲線と 3D 曲面の多項式表現とその応用)

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2D curve and 3D surface models are widely used for a variety of purposes in computer vision and graphics. There exist efficient function based shape representation techniques such as Fourier, B-Splines, Non-Uniform Rational B-Splines (NURBS), Radial Basis Function (RBF), Rational Gaussian and Implicit Polynomial (IP). Among these techniques, representation in IPs focuses attention from researchers in the vision community who, in particular, are developing sophisticated techniques for the automatic recognition, registration and matching of 2D/3D objects. In contrast to other representations, IPs are superior especially in the areas of fast fitting, compactness of parameters, algebraic/geometric invariants, robustness against noise and occlusion, etc.

IP representation mainly suffers from the following two issues that not only greatly limit its applicability, but also confuse the users who need to model their target objects with IPs:

1) Poor representation for complex shapes.

Objects obtained by vision modalities are often very precise and thus complex. Unfortunately, using one IP even with a high degree to model a complex object is nearly impossible, i.e., IP cannot give an accurate shape representation for a complex shape due to numerical instability and high computational cost.

2) Difficulty in determining a moderate IP degree and achieving global/local stability.

It often needs a fitting method for representing dataset with an IP. Prior IP fitting methods require that the degree of IP must be determined before handling fitting and therefore they are difficult to determine a moderate degree for a complex object. Furthermore the prior methods suffer from computational instability that many redundant zero-level sets are generated around the desired one. This makes the fitting result nearly impossible to be interpreted in the desired region space. The above two issues greatly limit the applicability of IP that the literatures, up to now, only applied IPs into such simple applications as single object recognition or global shape registration. This thesis makes three major contributions, two for solving these above two issues respectively and one for extending the IP's applicability for various applications.

#### - Contribution 1

In order to address issue 1), we propose a 3D segmentation method that is capable of dividing a complex 3D surface into simple surface segments and each segment of which can be accurately represented by an IP. This method is based on a cut-and-merge approach. Two cutting procedures adopt low-degree IPs to divide and fit the surface segments simultaneously, while avoiding generating high-curved segments. A merging procedure merges the similar adjacent segments to avoid over-segmentation. The purpose of this method aims at reducing the burden for encoding a complex object with an IP of high degree and simplifying the representation by encoding each simple segment with an IP of low degree.

The advantages of this method are that it is capable of i) finding the appropriate segmentation for various desired accuracies, ii) achieving stable fitting since we avoid using high-degree IPs, and iii) decomposing heavy computation from one high-degree IP into light computations from multiple low-degree IPs. This method maintains inherent algebraic/geometric invariants. Therefore the segmentation method opens up new vistas for 3D applications such as 3D matching, recognition and registration. This method can be applied to general 3D data formats such as polygonal mesh models.

#### - Contribution 2

Regarding issue 2), we propose a stable method for accurate fitting that automatically determines the moderate degree required. Our method increases the degree of IP until a satisfactory fitting result is obtained. The incrementability of the QR decomposition with Gram-Schmidt orthogonalization gives our method computational efficiency. Furthermore, since the decomposition detects the instability element precisely, our method can selectively apply ridge regression-based constraints only to that element.

The first advantage of this method is its computational efficiency because the incrementability of Gram-Schmidt QR decomposition dramatically reduces the burden of the incremental fitting by reusing the calculation results of the previous step. The second advantage is that the method can successfully avoid global instability, and furthermore maintain local accuracy, because constraints derived from the RR

regularization are selectively utilized in our incremental scheme. Finally, a set of stopping criteria can successfully stop the incremental scheme at the step where the moderate degree is achieved. Also as long as this method is properly applied into the above segmentation method, we obtain an extended version that the adaptive segmentation result is available.

#### - Contribution 3

In this contribution, we first exploit two new applications for 3D object matching and registration based on our segmentation method. For matching problem, we take advantage of the geometric invariants of IP which have been encoded into the each segment. Since these geometric invariants can be fast and easily extracted from the IP coefficients, 3D matching problem can be hence solved by finding the corresponding IPs on two segmented surfaces. For 3D registration problem, we utilize the property of IP that intrinsic center and principal axes of an IP are easily extracted from the coefficients. Therefore over the conventional methods our method achieves that 1) the registration is in a non-iterative process and thus fast; 2) no initialization is required.

Furthermore, based on the better IP representation obtained by our previous methods and for further extending the applicability of IP, we present a new and fast 2D-3D / 3D-3D alignment technique making use of 3D implicit polynomials (IP). The method registers a target object to a source object which has been modeled by IP in advance. In alignment process, our method drives the motion of IP toward the desired 2D/3D image features, such as object boundaries. The resulting evolution is the gradient flow derived from IP function that minimizes the proposed energy functional.

This method has been effectively applied into Ultrasound (US) image pose estimation for medical purposes. Ultrasound (US) images are widely used in clinic for diagnosis and image guidance. However, US images are notorious for the poor image quality, due to speckle noises, low signal-to-noise ratio, occlusions and uniform brightness. And field of view (FOV) in US imaging is very limited; in severe cases, only 2D ross-sectional images are obtained. Therefore, matching a 2D US image to a pre-operative 3D organ models is often desirable for medical diagnosis.

Three main advantages over the traditional registration method are i) a fast transformation method for IP coefficients is introduced; ii) the alignment is formulated as to minimize an energy functional derived from IP gradient flow and thus avoid the cost for calculating point-wise correspondences; iii) with the property of IPs having very few coefficients, it makes both IP gradients and its transformation calculation extremely light both on computational complexity and memory. Through applying our alignment techniques to a real-time US image pose estimation, we demonstrate the capabilities of overcoming the limitations of unconstrained freehand US data by robust and fast pose estimation.

In conclusion, the research provides insights into the theory and practice of polynomial representation, and the main contribution can be summarized by the three following points: Firstly, a 3D IP-segment representation method is developed even for robustly representing complex objects. Secondly, an efficient fitting method has been developed for adaptively estimating the IP of moderate degree for objects with various shapes. Thirdly, two applications of solving 3D matching and registration problem based on our segmentation method, and one registration method using IP gradients for medical image applications have been developed.