: Bayesian Analyses of Multinomial Tobit Models

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In recent years, Bayesian statistics has been popular in statistics since the Gibbs sampler, one of Markov chain Monte Carlo (MCMC) methods, is introduced as a sampling scheme for the posterior distribution. From a Bayesian point of view, statistical inference on model parameters are conducted on their posterior distribution. It is sometimes the case that this posterior distribution is a nonstandard one and is difficult for us to make statistical inferences. Thus, the classical Bayesian approach mainly focuses on models with so-called natural conjugate prior distribution where the posterior distribution is mathematically tractable. Such situation had been totally changed since MCMC methods, especially the Gibbs sampler and Metropolis-Hastings (MH) method, have been brought into this area. MCMC technique with the development of computing power makes it possible to analyze virtually any posterior distributions, and comprehensive researches are carried out to examine many statistical problems under the light of Bayesian statistics.

When we turn to econometrics, many statistical models used in this field, such as discrete choice and censored regression models, are also analyzed by Bayesian approach with MCMC. One advantage of Bayesian approach in econometrics is its flexibility to complex models. For the case of multinomial Tobit models described in later chapters, it is necessary for us to evaluate as many linear inequality constraints as observations when we estimate model parameters. Such inequalities are required for model parameter space being exclusively separated. Classical statistical procedures, such as the maximum likelihood estimation, however, are difficult to evaluate all these constraints in practice. Bayesian approach, on the other hand, can manage such situation by virtue of MCMC. Thus, this thesis takes Bayesian approaches to analyze multinomial Tobit models.

Tobit model is one of the most popular statistical models in econometrics and there are various extensions of the standard Tobit model. In terms of their form of likelihood functions, these models are classified into five types: from Type I to V Tobit model. Tobit model corresponds to various models in economics, and one of the most important applications is the model that includes corner solutions. When the economic model includes a point, for example 0, as one of optimal choices of an economic agent, its analogous statistical model has the positive probability on such point, and becomes one of five Tobit models. Multinomial Tobit models described in this thesis also have such corner solution models as their underlying economic models.

This thesis proposes Bayesian analyses of four multinomial Tobit models and one technical issue on a random number simulator often used in Bayesian econometrics. Empirical analyses are conducted for three of four Tobit models. Organization of this thesis is as follows.

In Chapter 2, we propose a Bayesian estimation method of demand functions under block rate pricing, focusing on increasing one. Under this pricing structure, price changes when consumption exceeds a certain threshold and the consumer faces a utility maximization problem subject to a piecewise-linear budget constraint. We apply the so-called discrete/continuous choice approach to derive the corresponding demand function. Then, its statistical model becomes a multinomial extension of Type V Tobit model. Taking a hierarchical Bayesian approach, we implement a Markov chain Monte Carlo simulation to estimate the demand function. Moreover, a separability condition is explicitly considered to obtain proper estimates. We find, however, that the convergence of the distribution of simulated samples to the posterior distribution is slow, requiring an additional scale transformation step for parameters to the Gibbs sampler. The model is also extended to allow random coefficients for panel data and spatial correlation for spatial data. These proposed methods are applied to estimate the Japanese residential water and electricity demand function.

Chapter 3 discusses a novel Bayesian estimation method for the residential gas demand function in Japan where the price per unit decreases as the demand exceeds certain thresholds. Such a price system is known as decreasing block rate pricing. The demand function under decreasing block rate pricing is derived by using the well-known discrete/continuous choice approach. However, because of the nonconvex budget set, the conventional approach imposes highly nonlinear constraints on the model parameters, thus making the maximization of the likelihood function under such constraints difficult to implement. To overcome this difficulty, we first apply the duality relationship in consumer theory, and approximate the conditional expenditure in order to linearize these nonlinear constraints. This linearized model becomes a multinomial extension of Type II Tobit model. Then, we adopt a Bayesian approach with the Markov chain Monte Carlo simulation in order to estimate the model parameters under linear constraints. Our proposed method is illustrated by a numerical example and is adopted to analyze the demand for residential gas in Japan.

Combining above two analyses, Chapter 4 proposes a multivariate modeling of the residential energy expenditure and its Bayesian estimation method. Residential energy resources, such as electricity and gas, are mainly provided under block rate pricing and the investigation on expenditure for energy resources has an important role in environmental policy makings. Nonlinear price schedules, however, prevented us from analyzing the residential energy demand. Previous chapters have shown that it is possible to examine single good's demand under block rate pricing. Thus, this chapter proposes a multivariate energy expenditure model under block rate pricing based on the discussion of single good's demand, and takes a Bayesian approach with Markov chain Monte Carlo method to estimate model parameters. Simulation and empirical analyses are a future work.

In Chapter 5, we focus on the friction model, a trinomial Tobit model. Friction model is one of the censored regression with censoring in small values and observing only large values.

Previously, these censoring limits are assumed to be constant. This chapter extends the classical friction model to allow covariate dependent thresholds and, by adopting MCMC, proposes a Bayesian estimation method. Our procedure is illustrated by a numerical example and applied to analyze prime rate data in Japan.

Chapter 6 reviews and compares four simulators that generate random variates following the multivariate normal distribution subject to linear constraints. Such random numbers are often required when we conduct Bayesian statistical inferences. There are mainly two types of simulators to draw these random variates: one that applies the Gibbs sampler and the other that adopts Metropolis-Hastings method. Because simulators that utilize MH method jointly draw samples from the desired distribution, obtained sample moves freely in its state space and shows quick decay of its sample autocorrelation. In particular, numerical examples reveal that Block Sampler, a simulator with the recursively truncated multivariate normal distribution as its proposal, shows the best performance among these four simulators.