博士論文の要旨

論文題目：Topological String Theory on Toric Calabi-Yau Manifolds and Instanton Counting

(トーリック カラビ-ヤウ多様体上の位相的弦理論とインスタントンの数え上げ)

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In this thesis, we study the instanton counting from the string and gauge theory perspective. The basic tools we use in this thesis are the Nekrasov partition function of instanton counting and the topological string theory on the toric Calabi-Yau. The Nekrasov theory of instanton counting describes the low energy dynamics of the $\mathcal{N} = 2$ gauge theory. The topological string theory on the toric Calabi-Yau capture the effective dynamics of the gauge theory as a local model of more complicated compactification of Type II superstrings. One of the main topics of this thesis is the relation of these gauge and string theories.

In the first part of this thesis, we have applied refined topological vertex for $SU(N)$ geometries and we compare the Nekrasov's theory of instanton counting. Then we show that the refined partition functions coincides with the the Nekrasov's partition functions in the non self-dual graviphoton background. The result has several meanings. First we show the validity of the attempt to refine the topological string theory by establishing the geometric engineering in the presence of the non self-dual background. This is a first step toward the formulation of the refined topological string theory in the general framework. It is also interesting that the Nekrasov partition function has the combinatorial interpretation using the Schur and Macdonald functions.

As we discussed in this paper, the refinement of the framing factor is obtained by computing the partition function for the SU(N) geometry. Since we cannot define the factor from the first principle at the present time, we derive it by comparing the refined topological vertex for $SU(N)$ geometry and the Nekrasov's partition function. If the refined topological string theory is formulated, it would reproduce

the factor we proposed in this thesis.

We extend the above correspondence to the gauge theories with matter hypermultiplets by studying the refined vertex on the strip geometry. We show that the refined vertex on the strip geometry is also put in the simple equation. This is because that the Schur functions of the partition functions can be summed up as in the case of the topological vertex on strips. Thus we find that the geometric engineering is extended to the gauge theory with the non-self dual graviphoton background. We can interpret this result as the reasonableness of attempt to refine the topological strings.

In the next part, we prove the flop invariance of the reined topological vertex. Since the flop invariance is the property which we expect to the refinement, it is very important to check the non-trivial property. We confirm the flop invariance by evaluating the flop amplitudes using the free fermions. Using the free fermions, we can represent these amplitudes as the Veneziano amplitute. This perspective is the background of the geometric engineering in the previous section. We also show the fundamental formula which is important to caluculate the Schur functions appearing in the refined vertex.

The results we have shown above are the studies of the refine vertex formalism. Then we study the mathematical applications. We compare the open string partition function of the conifold with the $sl(N)$ homological link invariants of the Hopf link. We can see the agreement between them for some small representations. Therefore we propose the superpolynomial as the conifold partition function. We expect that our proposal provides some insights into the study of the homological link invariants.

In the last part of this article, we apply the instanton counting to $\mathcal{N}=1$ gauge theories. Since the instanton counting in the $\mathcal{N}=2$ gauge theory is well studied, it is very important to apply this formalism to $\mathcal{N}=1$ theories. We compute the effective superpotentials from the microscopic superpotential formalism of $\mathcal{N}=1$ gauge theories which is obtained from the instanton counting technique. The essential point of this approach is same as the strong coupling approach which deals $\mathcal{N}=1$ theories as a deformation of the strongly coupled $\mathcal{N}=2$ theories . However the microscopic superpotential for the factorized curves give the useful way to study the vacua and phases of the theory systematically. Then we study the generalized glueball operators using this approach and the connection between the glueball operators, the Konishi anomaly equation, and the Witham hierarchy. We show that the Witham deformation which plays a key role in the Seiberg-Witten theory is also important in $\mathcal{N}=1$ theories.