論文内容の要旨

A Study of Finite-Size Effects in AdS/CFT Correspondence

(AdS/CFT 対応における有限サイズ効果の研究)

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It has been more than ten years since the seminal work by Maldacena, called the AdS/CFT correspondence. This correspondence is a realization of the gauge/string duality initially proposed by 't Hooft. He pointed out that the perturbative expansion of the $SU(N)$ gauge theory can be interpreted as a genus expansion of the string theory in the large N limit. Maldacena found the conjecture that certain conformal field theories (CFT) are dual to string theories on Anti-de Sitter (AdS) spaces times compact manifolds in the large N limit. In particular, the four-dimensional $\mathcal{N} = 4$ super Yang-Mills theory is dual to the type IIB superstring theory on $AdS_5 \times S^5$. The AdS/CFT correspondence is a strong/weak type duality, due to which it is hard to check (or prove) the correspondence.

In order to test the AdS/CFT correspondence quantitatively, we need to know the correspondence of physical quantities in the both theories. Fortunately there is a dictionary to relate quantities in one theory to those in the other. According to this dictionary, the energy of a string excitation, for example, corresponds to the conformal dimension, which is the eigenvalue of dilatation operator, of the dual gauge invariant operator. Thus the comparison of spectrum of both theories sheds light on checking the AdS/CFT correspondence.

In 2002, Minahan and Zarembo found that the dilatation operator at one-loop in the 't Hooft coupling is equivallent to an integrable quantum spin-chain. By virtue of their discovery, we can diagonalize the dilatation operator systematically

by using the so-called Bethe ansatz procedure. Since the perturbative dilatation operator contains long-range interactions, the Bethe ansatz to diagonalize it is called the asymptotic Bethe ansatz. Remarkably, the asymptotic Bethe ansatz equations producing all-loop spectrum have recently been proposed.

Though the asymptotic Bethe ansatz is very successful in investigating the perturbative spectrum of the dilatation operator, one should keep in mind that this method is broken down at the loop-order when long-range interactions exceed the length of the spin-chain (or operator). This finite-size effect is called wrapping effect. The similar effect is observed on string theory side due to virtual particles wrapping the worldsheet cylinder. The string Bethe ansatz also can not capture this effect. Thus to capture the wrapping effect, we need to find new fashions.

In this thesis, we study finite-size effects appearing in the AdS/CFT correspondence. Understanding the finite-size corrections is an important task and the first step towards the exact spectrum in finite-volume system. We discuss the finite-size effects for two cases: so-called dyonic giant magnons and multi giant magnons. For the dyonic giant magnons, we compute the leading finite-size correction to the energy in two ways. One is using the explicit string solutions, and the other is using the field theoretic tool called Lüscher formula. We confirm that both computations give the same result, and show that the finite-size dyonic giant magnons receive exponentially suppressed corrections. On the other hand, for the multi giant magnons, we propose the Lüscher formula for multi-particle states, and confirm it through some consistency checks. Using the proposed formula, we compute the finite-size energy shifts of the multi giant magnons at classical and one-loop levels. We find that our proposed formula reproduces the known results.