論文の内容の要旨

論文題目　Combinatorial Algorithms for Generalized Matching Problems
(マッチング問題の一般化に対する組合せ的アルゴリズム)

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The matching problem is the most fundamental combinatorial optimization problem solved in polynomial time. In the book *Matching Theory*, Lovász and Plummer say:

> Matching theory serves as an archetypal example of how a "well-solvable" problem can be studied. It also seems to be very near the limits of well-solvability in a sense. Most reasonable generalizations of the matching problem have turned out to be difficult---usually NP-hard---problems.

In fact, the outstanding works for the matching problem by Edmonds in the 1960s pioneered a new field of discrete mathematics, called polyhedral combinatorics. His results have formed a basis of study on combinatorial optimization up to the present time.

The main topics of this thesis are two generalizations of the matching problem. One is the independent even factor problem, which is a common generalization of the matching and matroid intersection problems. The other is the $K_{t,t}$-free t-matching problem in bipartite graphs, which is a generalization of the matching problem and a relaxation of the Hamiltonian cycle problem in bipartite graphs.

Both of these two problems are NP-hard in the most general settings. Under certain natural assumptions, however, these two problems become tractable. Important theorems for the matching problem such as a dual integrality theorem (the Cunningham-Marsh formula) and a decomposition theorem (the Edmonds-Gallai decomposition) are extended to the even factor problem in odd-cycle-symmetric digraphs. A dual integrality theorem for the $K_{t,t}$-free

t-matching problem is also established for bipartite graphs whose edge-weight is vertex-induced on any $K_{t,t}$. The two problems can be solved in polynomial time by the ellipsoid method under these assumptions.

From the viewpoint of designing combinatorial algorithms for the two problems, Pap's work was a breakthrough. For the unweighted cases, he succeeded in extending Edmonds' matching algorithm to the even factor problem in odd-cycle-symmetric digraphs and the $K_{t,t}$-free t-matching problem in bipartite graphs.

In this thesis, we extend Pap's work to weighted independent even factors and to weighted $K_{t,t}$-free t-matchings. That is, we devise combinatorial algorithms for the weighted independent even factor problem in odd-cycle-symmetric weighted digraphs and for the weighted $K_{t,t}$-free t-matching problem in bipartite graphs whose edge-weight is vertex-induced on any $K_{t,t}$. Our approach is to extend classical combinatorial algorithms such as the weighted matching algorithm, the weighted matroid intersection algorithm, and the minimum-cost flow algorithm. Our algorithms provide constructive proofs for the dual integrality and decomposition theorems. We also claim that the assumption of odd-cycle-symmetry for the even factor problem is reasonable, by showing that it is a necessary and sufficient condition for the even factors to induce a jump system.