論文の内容の要旨

論文題目 Differential Geometric Analysis for Dynamic Systems with Matrix-Valued States and Its Applications to Controlled Quantum Systems
（行列値状態を持つ動的システムに対する微分幾何的解析とその量子制御系への応用）

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Rapid miniaturization of electronic devices or theoretical development of quantum technologies demand control methodologies for quantum systems. Feedback control utilizing the continuous quantum measurement, so-called measurement-based quantum feedback control was proposed in current form in 1990s. Afterward, it has been extensively investigated both theoretically and experimentally because feedback control is anticipated to have advantages such as robustness against noise or modeling error as in the classical case. Thus, measurement-based quantum feedback control is regarded as one of the most promising control methods for quantum systems.

However, the dynamics of the controlled quantum system under continuous quantum measurement is described by a highly complicated equation and thus the properties of the system are difficult to capture. Especially, from the viewpoint of accessibility and controllability which are the concepts of control theory to evaluate the degree of freedom of state transition, analysis for the controlled quantum systems under continuous quantum measurement up to this time is insufficient. For instance, properties of controlled quantum systems with imperfect detector efficiency are little-known, and, to the best of the author's knowledge, the accessible set under control has not been revealed.

The main purpose of this dissertation is to minutely investigate the properties of controlled quantum systems under continuous quantum measurement from the viewpoint of accessibility. To this end, we develop a new method appropriate to tackle the problem, focusing on the fact that the state of a quantum system is described by a matrix.

In the anterior part of this dissertation, theory and techniques of differential geometric analysis for a class of dynamic systems with matrix-valued states are developed. The formulation is obtained by generalizing controlled quantum systems under continuous quantum measurement, and thus the developed method can be directly used to investigate accessibility of the controlled quantum systems. Lie product of matrix functions is introduced and plays an important role in the theory. A simple and coordinate-free calculation method for Lie product of matrix functions is derived, which is used as a powerful tool for the accessibility analysis. The theory of observability analysis is also developed.

In the posterior part of this dissertation, we apply the method developed in the anterior half to the analysis of the controlled quantum systems under continuous quantum measurement. The simple and coordinate-free calculation method for Lie product of matrix functions enables analysis with a clear perspective. We investigate the controlled quantum systems under continuous quantum measurement more deeply than the existing works, utilizing the notion of strong accessibility distribution. As a result, we derive a condition for measurement under which the local state transition of the system is quite limited. We also provide a theorem which can be a guideline for designing or choosing an actuator from a viewpoint of controlling the degree of freedom of the local state transition of the system. These are applicable to the systems with imperfect detector efficiency. Special phenomena which occur with perfect detector efficiency are also revealed. In addition, we derive an explicit form of the accessible set under certain conditions. To the author's best knowledge, this is the first result showing the explicit form of the accessible set of a controlled quantum system under continuous quantum measurement, and this can be also a guideline for designing or choosing an actuator. Among all the results derived in this dissertation, the two results which can be guidelines to determine an actuator have particular importance from the engineering point of view.

At the last part of the dissertation, further generalized formulation, namely, dynamic systems with affine state space is introduced and methods for accessibility and observability analysis are developed.