論文内容の要旨

Edge states, energy gap, and topological phases of graphene

(グラフェンのエッジ状態、エネルギーギャップ、トポロジカル相)

氏名：　江崎　健太

Motivated by recent experiments of graphene, such as quantum Hall effects near zero energy with half-filling in a magnetic field and a uniaxial strain in graphene, we examine the tight-binding model of the honeycomb lattice in the following three aspects:

(A) Anisotropy of the hopping integrals in a magnetic field.

(B) Incommensurate effects between the lattice and the magnetic field.

(C) Topological stability of the edge states under non-Hermiticity.

(A) First, we perform systematic study of states with zero energy (zero modes) and energy gaps by introducing effects of anisotropy t of hopping integrals for a tight-binding model on the honeycomb lattice in a magnetic field $2\pi\Phi = 2\pi p/q$ (p and q are mutually prime integers) as illustrated in Fig. 1. The condition for the existence of zero modes is analytically derived: Zero modes exist for $t \leq 2^{1/q}$, and a gap around zero energy opens for $t > 2^{1/q}$. For $t < 2$, a gap ΔE around zero energy in a weak magnetic field behaves as a non-perturbative and exponential form as a function of the magnetic field: $\Delta E \sim \exp(-\alpha/\Phi)$ with constants α depending on t as shown in Fig. 2. The non-perturbative behavior with respect to the magnetic field can be understood as tunneling effects between energy levels around two zero modes with linear dispersion (Dirac zero modes) appearing in the honeycomb lattice, and for $t \sim 2$ an explicit form of the gap around zero energy is obtained by the WKB method near the merging

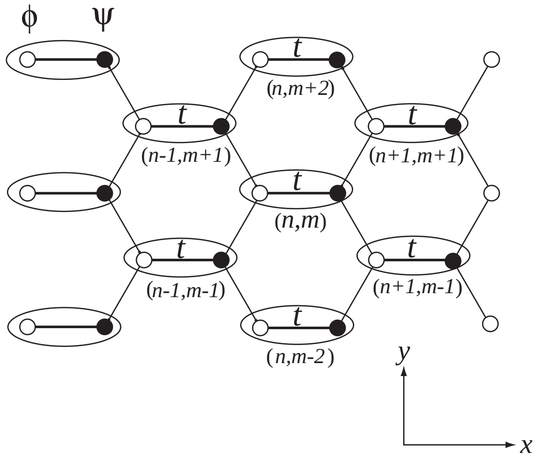

Figure 1: The honeycomb lattice. The hopping integrals of the horizontal bonds are t, and those for the other bonds are 1. A magnetic flux $2\pi\Phi$ is applied through the unit hexagon.

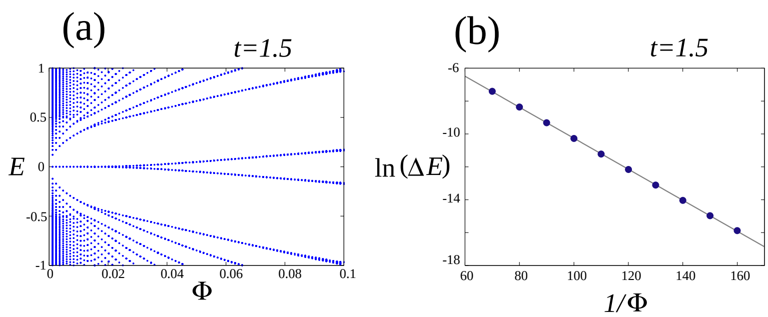

Figure 2: (a) Energy bands as a function of Φ for $t = 1.5$. (b) The natural logarithm of the gap ΔE around $E = 0$ as a function of $1/\Phi$ for $t = 1.5$.

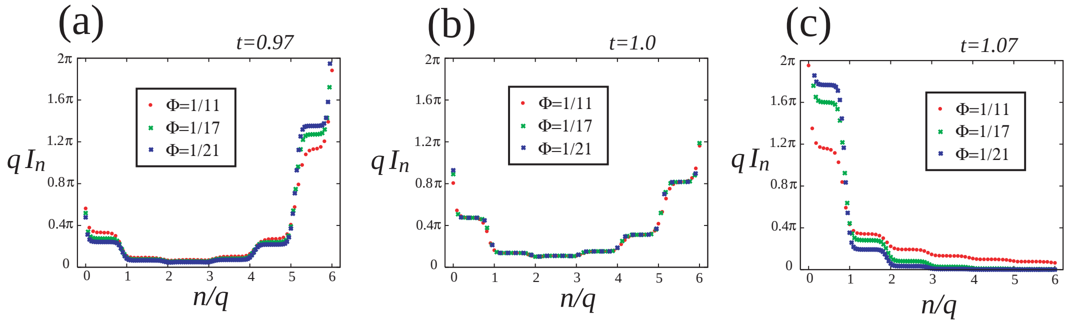

Figure 3: Scaled plots of local charge densities qI_n as functions of coordinates n/q for $\Phi = 1/11$, $1/17$, and $1/21$. (a) $t = 0.97$, (b) $t = 1.0$, (c) $t = 1.07$.

point of these Dirac zero modes as

$$\Delta E \simeq 4\sqrt{\Phi G}\exp\left(-\frac{2G^3}{3\pi\Phi}\right), \quad G = \sqrt{2-t}. \tag{1}$$

Effects of the anisotropy for the honeycomb lattices with boundaries are also studied. The condition for the existence of zero energy edge states in a magnetic field is analytically derived. On the basis of the condition, it is recognized that anisotropy of the hopping integrals induces abrupt changes of the number of zero energy edge states, which depend on the shapes of the edges sensitively (see Fig. 3).

(B) We also study incommensurate effects between the lattice and the magnetic field, which are realized by taking the magnetic flux Φ to be an irrational number. From the examination of the sum of the band widths and multifractal analysis of energy spectra, a phase diagram of a localization problem is proposed as Fig. 4, where the values of the hopping integrals t are varied. Especially, the graphene (the isotropic honeycomb lattice) is found to correspond to a phase transition point of the localization problem. The proposed phase diagram is also supported by multifractal analysis of wavefunctions. At the same time, specialty of zero energy states is rediscovered for the graphene case.

(C) In addition to the above-mentioned Hermitian systems, we also consider non-Hermitian systems. Topological stability of the edge states is examined in the presence of decay of wavefunctions or asymmetry of the hopping integrals due to non-Hermiticity. For several non-Hermitian extensions of spin-Hall systems, gapless edge states are found under small non-Hermiticity. They are topologically protected by spin Chern numbers for time-reversal operators Θ with $\Theta^2 = +1$ (see Fig. 5). We also examine graphene with decay of wavefunctions. Edge states with Re$E = 0$ appear in this model, and their topological stability is explained by winding numbers.

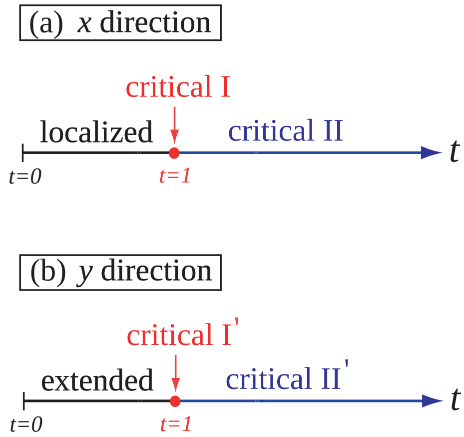

Figure 4: Proposed phase diagram for quasiperiodic systems obtained from honeycomb lattices in an irrational magnetic flux. For $0 < t < 1$, states are localized in the x direction and extended in the y direction. In critical I (I') phase for $t = 1$ and critical II (II') phase for $t > 1$, energy spectra and wave functions are multifractal.

According to multifractal analysis of energy spectra, the critical I (I') phase for $t = 1$ is proposed to belong to the same universality class as that of the isotropic triangular lattice. On the other hand, the critical II (II') phase for $t > 1$ is proposed to belong to the same universality class as that of the isotropic square lattice.

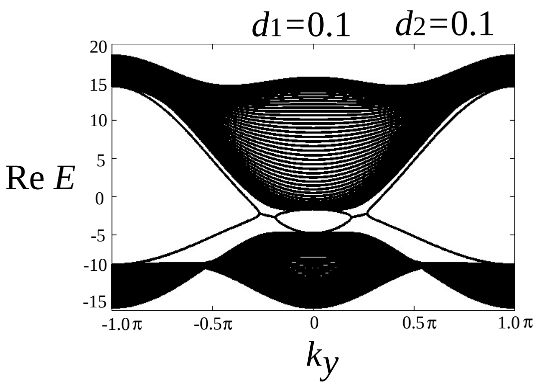

Figure 5: Gapless edge states brought by spin Chern numbers for time-reversal operators Θ with $\Theta^2 = +1$ in non-Hermitian systems.