論文内容の要旨

論文題目：Dirac spectra in dense QCD
（高密度 QCD におけるディラック固有値の研究）

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In this thesis I will study characteristics of the Dirac spectrum in QCD at high baryon density. I will combine the low energy effective theory of Nambu-Goldstone (NG) bosons and chiral random matrix theory (ChRMT) to investigate how the BCS pairing of quarks and the resulting symmetry breaking show up in the distribution of low-lying Dirac eigenvalues.

I first consider the ε-regime of the color-flavor-locked (CFL) phase of QCD for three flavors at high density, where the dynamics is dominated by zero modes of NG bosons. Comparing the ultraviolet expression of the finite-volume partition function in terms of Dirac eigenvalues with that coming from the low-energy effective theory near the Fermi surface, I will derive Leutwyler-Smilga-type spectral sum rules for Dirac eigenvalues and show that the BCS gap is the central quantity which characterizes the distribution of low-lying Dirac eigenvalues at high density. Next, I consider QCD with gauge group SU(2) (called two-color QCD) at high density, motivated by the fact that a first-principle lattice simulation is feasible in two-color QCD even at nonzero baryon density. The symmetry breaking pattern caused by a diquark condensate is discussed, in comparison with that at low density, and a low-energy effective theory for emergent NG modes is constructed with nonzero current quark mass. In the ε-regime I compute the quark mass dependence of the partition function exactly and derive novel spectral sum rules for Dirac eigenvalues at high density. I also define the microscopic limit of the spectral density at high density, and argue that it should be universal, i.e., solely determined by global symmetries and insensitive to the ultraviolet dynamics of QCD.

Inspired by the success of ChRMT at zero density, I introduce a new random matrix ensemble which shares the same symmetries as two-color QCD at high density. Within a saddle-point approximation I show that the large-N limit of the matrix model is identical to the ε-regime partition function at high density. The model is solved at large N in both limits of weak and strong non-Hermiticity, and the microscopic spectral density is obtained exactly for general number of flavors. Based on this, the severity of the sign problem is analyzed at low and high density, through the average sign of the quark determinant.

I will also discuss the insertion of the diquark source term into the effective theory and ChRMT. After deriving a high density analogue of the Smilga-Stern relation and new spectral sum rules for singular values of the Dirac operator, I will argue that it is the diquark condensate, rather than the BCS gap, which governs the distribution of small singular values. Finally I will generalize the Banks-Casher relation to the high density BCS phase, pointing out that the density of Dirac eigenvalues at the origin is proportional to the square of the BCS gap. All these analytical results can in principle be tested in future lattice simulations.