論文内容の要旨

論文題目　Thermodynamics of Information Processing in Small Systems
（微小系における情報処理の熱力学）

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We construct a general theory of thermodynamics that can be applied to information processing in small systems. In particular, we derive generalized formulas of the second law of thermodynamics that quantitatively reveal the fundamental lower bounds of the energy costs that are needed for information processing such as the feedback control, measurement, and information erasure. The obtained results include the terms of information contents as well as thermodynamic quantities. Moreover, we generalize the general equalities in nonequilibrium statistical mechanics to the situation in which a thermodynamics system is subject to feedback control. Our results are model independent, so that they can be applied to a broad class of information processing.

In terms of fundamental physics, our study is closely related to the foundation of thermodynamics and statistical mechanics; we discuss the resolution of the paradox of “Maxwell’s demon.” In fact, from the modern point of view, Maxwell’s demon can be formulated as a feedback controller acting on a thermodynamic system. Moreover, our results serve as a theoretical foundation for the control of small systems both in classical and quantum regimes.

Historically, J. C. Maxwell mentioned his gedankenexperiment of the demon in 1867 for the first time, and later on, a more quantitative model of the demon was proposed by L. Szilard in 1929. In the gedankenexperiment by Szilard, the demon performs a measurement on the position of a single-molecule gas and gets one bit ($= \ln 2$ nat) of information. By using the obtained information, the demon can extract $k_B T \ln 2$ of work from a single heat bath during a thermodynamic cycle, where k_B is the Boltzmann constant and T is the temperature of the heat bath. This apparent contradiction against the second law of thermodynamics has been attracted a lot of attentions of numerous researchers. Since the 1980's, it has been widely accepted that the information erasure process from demon's memory plays the key role to resolve the paradox of Maxwell's demon. However, we show that this resolution is not complete, and propose the new resolution of the paradox. We note that the Szilard-type Maxwell's demon has recently been realized by a colloidal particle in a real experiment.

Recently, Maxwell's demon has been attracted renewed attentions. One of the important backgrounds lies in the fact that thermodynamic aspects of small systems have become more and more important. The crucial feature of such small thermodynamic systems is that their dynamics is stochastic, so that their thermal fluctuations become the same order of magnitude as the averages of the physical quantities. Therefore, the fluctuations play crucial roles to understand the dynamics of such systems. We note that, in small systems, the second law of thermodynamics is stochastically violated due to thermal fluctuations, while it has been established that the second law still holds on average.

Experimentally, due to the recent development of nanoscience and nanotechnology, one can control and measure small thermodynamic systems more precisely than the level of thermal fluctuations in both classical and quantum regimes. In the classical regime, one can manipulate a single macromolecule or a colloidal particle at room temperature, by using, for example, optical tweezers. This technique has been applied to investigate biological systems such as the molecular motors such as Kinesins and F1-ATPases. Moreover, artificial molecular machines have also been a topic of active researches. In the quantum regime, one can perform quantum measurement and control at the level of a single atom or a single photon with quantum coherence, where quantum dots can be regarded as a typical example. The technologies of quantum measurement and control have been applied to quantum information theory.

Theoretically, the study in this thesis is based on the recently-developed theories of nonequilibrium statistical mechanics and quantum measurement and information theory.

In nonequilibrium statistical mechanics, a lot of equalities that are universally valid in nonequilibrium stochastic systems have been found since 1993, and they have been experimentally verified in small thermodynamic systems. A prominent result is the fluctuation theorem, which enables us to quantitatively characterize the probability of the stochastic violation of the second law of thermodynamics in small systems. Another prominent result is the Jarzynski equality, which expresses the second law of thermodynamics by an equality rather than an inequality.

On the other hand, quantum measurement and information theory has been established, and has been applied to a lot of experimental situations including quantum-optical systems. By using positive operator-valued measures (POVMs) and measurement operators, one can quantitatively calculate the probability distributions of the outcomes and backactions of quantum measurements. Moreover, the important concepts in classical information theory have been generalized to quantum information theory. The crucial concepts in information theory are the Shannon information and the mutual information. The former characterizes the randomness of the measurement outcomes, while the latter characterizes the correlation between the measured state and the measurement outcomes. The mutual information represents the effective information that is obtained by the measurement, and equals the Shannon information if the measurement is error-free.

The main results of this thesis consist of three parts as follows.

First, we generalize the second law of thermodynamics to the situations in which small thermodynamic systems are subject to quantum feedback control. For the special case in which there is a single heat bath at temperature T, the generalized second law can be expressed as

$$W_{\mathrm{ext}}^{\mathrm{S}} \leq -\Delta F^{\mathrm{S}} + k_{\mathrm{B}} T I_{\mathrm{QC}},$$

where $W_{\mathrm{ext}}^{\mathrm{S}}$ is the work that is extracted by the demon, ΔF^{S} is the free-energy difference of the system, and I_{QC} is a quantum extension of the mutual information that is obtained by the measurement. Our result leads to the fundamental upper bound of the work that can be extracted by the demon.

Second, we generalize the second law of thermodynamics to the measurement and information erasure processes of the memory of the demon. In fact, the demon (or the feedback controller) has a memory that stores measurement outcomes. The generalized second laws identify the lower bounds of the energy costs that are needed for the measurement and information erasure.

The minimal energy cost for the measurement is determined as

$$W_{\mathrm{meas}}^{\mathrm{M}} \geq -k_{\mathrm{B}} T (H - I_{\mathrm{QC}}) + \Delta F_{\mathrm{meas}}^{\mathrm{M}},$$

where $W_{\mathrm{meas}}^{\mathrm{M}}$ is the work that is performed on the memory, H is the Shannon information, and $\Delta F_{\mathrm{meas}}^{\mathrm{M}}$ is the free-energy change in the memory during the measurement. On the other hand, the minimal energy cost for the erasure is determined as

$$W_{\mathrm{eras}}^{\mathrm{M}} \geq k_{\mathrm{B}} T H + \Delta F_{\mathrm{eras}}^{\mathrm{M}},$$

where $W_{\mathrm{eras}}^{\mathrm{M}}$ is the work that is performed on the memory, and $\Delta F_{\mathrm{eras}}^{\mathrm{M}}$ is the free-energy change in the memory during the erasure. If the free-energy change is zero, the minimal cost for the erasure leads to the celebrated Landauer's principle. By summing up the costs for the measurement and erasure, we obtain the trade-off relation:

$$W_{\mathrm{meas}}^{\mathrm{M}} + W_{\mathrm{eras}}^{\mathrm{M}} \geq k_{\mathrm{B}} T I_{\mathrm{QC}},$$

where the lower bound is determined only by the mutual information.

Based on the obtained three inequalities, we can reconcile Maxwell's demon with the second law of thermodynamics. The conventional second law of thermodynamics is shown to be satisfied for the total system of the measured system and demon's memory. What reconciles the demon with the second law is the total work of the measurement and erasure, which compensates for the excess work that can be extracted by the demon.

We derive the aforementioned inequalities based on quantum-statistical mechanics and quantum measurement theory. However, the obtained result can also be applied to classical systems as the classical limits.

Third, we generalize nonequilibrium equalities, such as the fluctuation theorem and the Jarzynski equality, to classical stochastic dynamics in the presence of feedback control. We derive two types of generalizations; one involves the term of the information obtained by measurements, and the other involves the term that characterizes the efficacy of feedback control. One of the generalized equalities has recently been experimentally verified with the feedback control of a colloidal particle.

In our results, thermodynamic quantities and information contents are treated on an equal footing. Moreover, the obtained inequalities reduce to the conventional second law of thermodynamics for a special case in which all of the information contents are zero. Therefore, our results can be regarded as constituting the second law of "information thermodynamics."