論文内容の要旨

Characterizing globalness of unitary operations for quantum information processing (量子情報処理におけるユニタリ演算の非局所性の解析)

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Quantum information science has shown that models of information processing devices following quantum principles can execute algorithms that outperform the best known algorithms designed to be executed on models following classical principles. The source of this extra-classical performance has been a perplexing problem as it has also been shown that some quantum models can be efficiently simulated classically. One such quantum model that can be simulated by a classical model is what we call the *LOCC model*, which makes use of a collection of two-level quantum systems (called qubits) and coordinated operations on individual qubits, a situation commonly called *local operations and classical communication* (LOCC). It is known that extra-classical performances can be achieved if multi-qubit unitary operations are added to the LOCC model, indicating that these operations must have certain properties that are essential for the gap in performance.

In general, there is a class of information processing tasks that cannot be accomplished when operations are restricted to LOCC. In this thesis, we refer to these tasks as *global tasks*, and the property of unitary operations that allow them to accomplish these tasks as *globalness.* The globalness of unitary operations is an important resource for realizing the extra-classical performance in quantum information processing, making it worthy of investigation.

In this thesis, we analyze globalness of unitary operations that are particularly relevant to global tasks dealing with unknown input states by studying three global tasks we call (i) *two-piece delocalization*, (ii) *one-piece delocalization*, and (iii) *entanglement-assisted LOCC implementation* of global unitary operations.

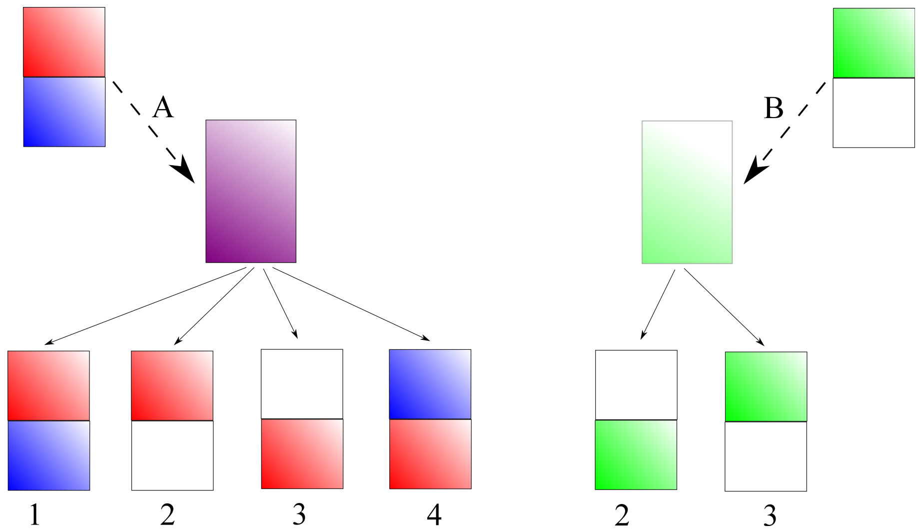

Figure 1: Schematics of delocalization, LOCC relocalization, and LOCC relocation. Each small colored box represents a qudit with one piece of quantum information (or, its corresponding Hilbert space, to be precise). Each unfilled small box represents a blank qudit. The dashed arrow A corresponds to two-piece delocalization, while B to one-piece delocalization. The solid arrows pointing down to each box on the bottom represent LOCC. Boxes 1, 2, 3, and 4 correspond to two-piece LOCC relocalization, one-piece LOCC relocalization, one-piece LOCC relocation, and two-piece LOCC relocation, respectively.

We begin by explaining our notion of *one piece of localized quantum information*, which is a state of a d-level quantum system (or *qudit*) that is guaranteed to be representible by a certain d-dimensional state vector, but the choice of the vector is totally unknown. It is also argued in this chapter why we refer to this state as *localized* quantum information. Global tasks (i) and (ii) are then introduced. We observe that there are 'degrees' of delocalization by considering four LOCC tasks we name *one-piece LOCC relocalization*, *two-piece LOCC relocalization*, *one-piece LOCC relocation*, and *two-piece LOCC relocation*. Roughly speaking, these LOCC tasks are used to measure how initially localized quantum information is moved out of its original Hilbert space. See Fig. 1 for a schematic representation of each task.

More precisely, we obtain the following three results when two-piece

LOCC relocalization and one-piece LOCC relocalization are used. First, we prove that two-piece LOCC relocalization is possible after two-piece delocalization, if and only if the delocalizing unitary operation is a tensor product of two local unitary operations. Second, we prove that one-piece LOCC relocalization is possible after two-piece delocalization, if and only if the delocalizing unitary operation is local unitarily equivalent to a controlled-unitary operation. Third, we restrict the delocalizaing unitary operation to be a two-qubit unitary operation and prove that one-piece LOCC relocalization is possible after one-piece delocalization if the given unitary operation has at most two nonzero Kraus-Cirac coefficients.

As for two-piece LOCC relocation and one-piece LOCC relocation, we obtain the following three results. First, two-piece LOCC relocation is possible after two-piece delocalization, if and only if the delocalizing unitary operation is local unitarily equivalent to the swap operation. Second, it is proven that if one-piece LOCC relocation is possible after one-piece delocalization, then the delocalizing unitary operation must be non-invertible under partial transposition. As a corollary, we show that local unitary equivalents of controlled-unitary operations do not give two pieces of delocalized quantum information that are one-piece LOCC relocateable. Finally, we give an example of a unitary operation that gives one piece of delocalized quantum information that is one-piece LOCC relocateable.

Before discussing how this analysis leads to a characterization of globalness of unitary operations, we analyze the third global task, entanglement-assisted LOCC implementation of *two-qubit* global unitary operations, to evaluate the entanglement resources necessary to deterministically implement any given controlled-unitary operation by LOCC. The minimal amount of entanglement measured in terms of LOCC monotones is obtained. It is proven that for any given two-qubit controlled-unitary operation, its deterministic entanglement-assisted LOCC implementation requires at least 1 ebit when the resource state is two-qubit. We derive conditions that any LOCC protocol must satisfy when it implements the given controlled-unitary operation. These conditions are used to show that any such protocol can be transformed without loss of generality to a three-turn protocol for which the necessary entanglement resource must be a maximally entangled two-qubit state. This answers a decade long open question in entanglement theory.

Based on these analyses of the three global tasks (i)-(iii), we argue that they naturally lead to characterizations of globalness. The first characterization is based on the delocalization power of global unitary operations. We present two ways to rank delocalization power of global unitary operation according to the degree of delocalization each unitary operation brings. The difference in these rankings lies in whether the degree of delocalization is measured by LOCC relocalization or LOCC relocation. The second characterization is based on the minimal entanglement cost required to perform a deterministic entanglement-assisted LOCC implementation for a given unitary operation. We study relations between these two characterizations.

Moreover, these characterizations, which are all based on gloal tasks on *unknown* input states, are compared against a characterization based on a global task on *known* input states. This characterization has been investigated elsewhere and is called *entangling power*, which is measured by the maximal amount of entanglement that a given unitary operation can generate. After proving a generic statement about entangling power, we find that entangling power is unrelated to the characterizations based on the global tasks on unknown input states. It is argued that the degree of globalness of unitary operations reflects the fundamental difference between the known and unknown input states in the task used to characterize the globalness.