論文の内容の要旨

Computational Complexities of Interior-Point Methods in Symmetric Cone Programs and Their Information Geometric Analyses
(対称錐計画問題に対する内点法の計算複雑度と情報幾何学的解析)

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An interior-point method (IPM) is a convex programming method which widely attracts many areas of research and development. This is because it exhibits theoretical certificates of polynomial complexities, wider class of applications, and superior computational capabilities in real world situations. In its early stage of development, it has been only concerned with linear programming (LP), but later it has been extended to larger class of problems such as semidefinite programming (SDP), second-order cone programming (SOCP), symmetric cone programs, etc. Simultaneously, a various types of algorithms has been developed; they are roughly classified based on the following three independent criteria: i) primal/dual algorithms or primal-dual algorithms, ii)path-following methods, potential reduction methods, or affine-scaling methods, iii) feasible methods or infeasible methods. On top of these subjects, complexity analysis has always been a central theme in IPMs, in fact polynomial complexity and its improvement are always in debate.

In this thesis, we first discuss iteration complexities of Mizuno-Todd-Ye predictor-corrector (MTY-PC) algorithm in view of the "curvature" of the central path, a well-defined trajectory in the feasible solutions, in case of SDP and symmetric cone programs. MTY-PC algorithm is a path-following primal-dual algorithm which traces the central path inside its neighborhood toward Newton direction at a predictor step and toward a central direction at a corrector step in alternative shift to reach an optimum solution.

The derivation of iteration complexities is based on the intuitive idea that path-following algorithms trace the straight parts of the central path faster than remaining curved parts. Our analysis is an asymptotic one with letting the opening of the neighborhood zero.

Second, we discuss the relationship of iteration complexities between path-following primal/dual algorithms and MTY-PC algorithm in view of information geometric framework recently developed by Ohara and Tsuchiya. Primal/dual algorithms are a general framework, established by Nesterov and Nemirovski, to deal with a conic linear programs including SDP and symmetric cone programs, and solve problems only in primal or dual feasible regions. Using information geometric framework, we can consider both a primal problem and a dual problem in the same space through Legendre transformation, Under this setting, we prove the Pythagorean relationships between them in case of SDP and symmetric cone programs based on the fact that a certain projection of the "curvature" of MTY-PC algorithm is the one of a primal algorithm and its orthogonal projection is the one of dual algorithm.

Lastly, we conduct reasonably large-scale numerical experiments in LP and SDP, so as to investigate the validity of the analyses of iteration complexities in MTY-PC algorithms for real problems. In spite of an asymptotic treatment of analysis by nature, numerical experiments strongly suggest our claim hold for practically large opening of the neighborhood.