論文内容の要旨

論文題目　Hydrodynamic Description of Spin-1 Bose-Einstein Condensates
（スピン１ボース・アインシュタイン凝縮体の流体力学的記述）

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We establish self-contained hydrodynamic equations for a spin-1 spinor Bose-Einstein condensate (BEC) for an arbitrary state including nonequilibrium one. The obtained hydrodynamic equations, which yield a description equivalent to the time-dependent multi-component Gross-Pitaevskii (GP) equation, involve the continuity equations for the density, the spin, and the nematic tensor (or the quadrupolar tensor) and the equation of motion for the density. These equations are written only in terms of observable physical quantities, i.e., the mass current in addition to the density, the spin, and the nematic tensor.

We derive the low-lying collective modes in the ferromagnetic (FM) state and the polar (P) state from our hydrodynamic equations: we apply linearization to the hydrodynamic equations with respect to the fluctuations of the density, the spin, and the nematic tensor. Then, the differential equations for them are derived, which yield the dispersion relations. The obtained dispersion relations are consistent with those calculated from the time-dependent multi-component GP equation or the Bogoliubov approximation. In addition, it is found that one of the spin modes in each state originates from the fluctuations of the nematic tensor.

Since the hydrodynamic equations are described by observable quantities, they help us to understand the physical properties of a spinor BEC intuitively. Moreover, in a spinor BEC system, the in-situ and high-resolution observation of the magnetization profile is possible, which augments the significance of the spinor hydrodynamics. We can elucidate and predict features of a spinor BECs, such as spin textures and their dynamics, in a physically more transparent manner in comparison to the time-dependent multi-component GP equation written in terms of spinor order parameters (or a spinor wave function), which cannot be detected directly.

A spinor BEC, which is a BEC with spin-internal degrees of freedom, shows a rich variety of phenomena originating from its spin-degrees of freedom, such as magnon collective modes, spin texture formation, and topological excitations. Many of these properties are well understood by several variations of the mean-field approximations, i.e., Bogoliubov-de-Gennes approximation and the multi-component GP equation developed by Ho and Ohmi & Macida.

Experimentally, a spinor BEC was first realized in a sodium-23 vapor confined in an optical trap by Davis *et. al.* in 1998. Prompted by the realization of spinor BECs, various experiments have been performed, for example, a BEC prepared in a metastable state, quantum tunneling of a BEC, and spin-exchange dynamics. Furthermore, Higbie *et. al.* developed an in situ and high-resolution imaging of magnetization profiles for spinor BECs, which prompted the theoretical researches of the hydrodynamic description for spnor BECs.

The pioneering works of the spinor hydrodynamics were done by Lamacraft and Kudo & Kawaguchi, which mainly focus on the experiment by Sadler *et. al.*, i,e., spin domain formation in a quenched ferromagnetic BEC of rubidium-87. According to their works, the obtained hydrodynamic equations for a ferromagnetic BEC are written in terms of observable quantities, that is to say, the density ρ, the mass current $\boldsymbol{v}_{\mathrm{mass}}$, and the spin density $\boldsymbol{f}$. The hydrodynamic equation reduces to a modified Landau-Lifshitz equation with the assumption of the spatially uniform density in the work of Lamacraft, whereas the hydrodynamic equation is expressed in the form of a Landau-Lifshitz-Gilbert equation in the work of Kudo and Kawaguchi. The hydrodynamic equation for the polar BEC is also derived by Kawaguchi, and there are a couple of formalisms that can be applied to general spins and states by means of the Majorana representation of spin states; however, the hydrodynamic equations that are written in terms of observable physical quantities and enable us to treat a spinor BEC in an arbitrary state have not been established yet.

Thus, in this thesis, we aim to develop such spinor hydrodynamic equations involving only physical quantities, which can apply to a BEC in an arbitrary state in the mean-field regime.

The main results of this thesis are comprised of three parts as follows.

Firstly, the appropriate variables that describe the hydrodynamic equations are determined on the basis of the degrees of freedom of the multi-component GP equation. Corresponding to the six variables of the multi-component GP equation, the hydrodynamic equations require the thirteen variables, that is to say, the density ρ, the mass current $\boldsymbol{v}_{\mathrm{mass}}$, the spin density $\boldsymbol{f}$, and the nematic tensor $N_{\mu\nu}$, where the nematic tensor indicates anisotropy and axes of a spinor wave function in the multi-component GP description. By introducing the nematic tensor, the hydrodynamic equations for a spin-1 BEC are generalized to arbitrary states, i.e., the FM state, the P state, and the broken-axisymmetry (BA) state, which has a broken axisymmetry about an external magnetic field.

Secondly, we derive the hydrodynamic equations in terms of the variables mentioned above. The hydrodynamic equations are made up by the continuity equations and the equation of motion.

The continuity equation for the density, which involves the time derivative of the density and the divergence of the mass current, is expressed by the same form as that for the FM state except for the explicit form of the mass current:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \boldsymbol{v}_{\mathrm{mass}} = 0, \qquad \boldsymbol{v}_{\mathrm{mass}} = \frac{\hbar}{M}\left[(\nabla\phi) - (\nabla\alpha) f_z - (\nabla\gamma)\sin 2\vartheta\right],$$

where M indicates the mass of a particle and α, β, and γ denote the angles of an Euler rotation of the spinor wave function, and ϑ expresses the state of the BEC. Here, as a consequence of the generalization of the mass current, the Mermin-Ho relation of the mass current for the FM state is also generalized so that the Berry phase changes by a factor of the magnitude of the spin $|\boldsymbol{f}|$, and the difference between the circulation of the mass current and the Berry phase is given by $(h/M)(n - 2|\boldsymbol{f}|)$, where n is an integer.

The continuity equation for the spin is also written in the same expression as that for the FM state except for the spin current $\boldsymbol{v}_\mu$:

$$\frac{\partial \rho f_\mu}{\partial t} + \nabla \cdot \rho \boldsymbol{v}_\mu = \frac{1}{\hbar}\epsilon_{z\mu\nu}\rho\left(p f_\nu - 2qN_{z\nu}\right), \qquad \boldsymbol{v}_\mu = f_\mu \boldsymbol{v}_{\mathrm{mass}} - \frac{\hbar}{M}\epsilon_{\mu\nu\lambda}\left[N_{\nu\eta}\left(\nabla N_{\lambda\eta}\right) + \frac{1}{4}f_\nu\left(\nabla f_\lambda\right)\right],$$

where the terms on the right-hand side originate from the linear and quadratic Zeeman terms. For an arbitrary state, the spin current involves not only terms originating from mass transport and a spatial profile of $\boldsymbol{f}$ but a term arising from spatial derivatives of the nematic tensor. For the particular case of the FM state, the expression of the spin current reduces to that for the FM state.

The continuity equation for the nematic tensor is obtained as

$$\frac{\partial \rho N_{\mu\nu}}{\partial t} + \nabla \cdot \rho \boldsymbol{v}_{\mu\nu} = \frac{1}{\hbar}\rho\left[\epsilon_{z\mu\lambda}\left(pN_{\nu\lambda} - \frac{q}{2}\delta_{z\nu}f_\lambda\right) + \epsilon_{z\nu\lambda}\left(pN_{\mu\lambda} - \frac{q}{2}\delta_{z\mu}f_\lambda\right)\right] + \frac{c_1}{\hbar}\rho^2\left(\epsilon_{\mu\lambda\eta}f_\lambda N_{\nu\eta} + \epsilon_{\nu\lambda\eta}f_\lambda N_{\mu\eta}\right),$$

$$\boldsymbol{v}_{\mu\nu} = N_{\mu\nu}\boldsymbol{v}_{\mathrm{mass}} - \frac{\hbar}{4M}\left\{\epsilon_{\mu\lambda\eta}\left[f_\lambda\left(\nabla N_{\nu\eta}\right) - \left(\nabla f_\lambda\right)N_{\nu\eta}\right] + \epsilon_{\nu\lambda\eta}\left[f_\lambda\left(\nabla N_{\mu\eta}\right) - \left(\nabla f_\lambda\right)N_{\mu\eta}\right]\right\},$$

where $\boldsymbol{v}_{\mu\nu}$ represents the nematic current. The term in the second line on the right-hand side of the continuity equation implies that the outer product of the spin and the column vector of the nematic tensor acts like a torque on the nematic tensor, which causes the nematic current as well as mass transport.

The last equation of the hydrodynamic description is the equation of motion for the density:

$$\begin{aligned}\frac{\partial \rho v_{\mathrm{mass},i}}{\partial t} =& \frac{\hbar^2}{2M^2}\rho\nabla_i\frac{\nabla_j^2\sqrt{\rho}}{\sqrt{\rho}} - \nabla_j\left(\rho v_{\mathrm{mass},i}v_{\mathrm{mass},j}\right)\\ &- \frac{\hbar^2}{4M^2}\nabla_j\Big\{\rho\Big[\left(\nabla_i N_{\mu\nu}\right)\left(\nabla_j N_{\mu\nu}\right) - \left(\nabla_i\nabla_j N_{\mu\nu}\right)N_{\mu\nu}\\ &+ \frac{1}{2}\left[\left(\nabla_i f_\mu\right)\left(\nabla_j f_\mu\right) - \left(\nabla_i\nabla_j f_\mu\right)f_\mu\right]\Big]\Big\}\\ &- \frac{1}{M}\left(\nabla_i U\right)\rho - \frac{c_0}{2M}\left(\nabla_i\rho^2\right) - \frac{c_1}{2M}\left[\nabla_i(\rho f_\mu)^2\right],\end{aligned}$$

which is analogous to the Euler equation. As we see, one of the features of a spinor BEC appears on the right-hand side of the equation of motion: there are the quantum pressure terms caused by not only the density but the spin and the nematic tensor.

Similarly, we also derive the equations of motion for the spin and the nematic tensor. Our hydrodynamic equations reduce to the hydrodynamic equations for the ferromagnetic BEC under the condition of the FM state.

Finally, we linearize the equations of motion for the density, spin, and the nematic tensor with respect to their fluctuation, and derive the dispersion relations for the low-lying collective modes in the FM state and the P state. The obtained results, which reproduce the elementary excitations calculated from the multi-component GP equation, imply that one of the magnons in each state is caused by the fluctuation of the nematic tensor.

In conclusion, we construct the hydrodynamic equations that are equivalent to the time-dependent multi-component GP equation. Our hydrodynamic equations are totally written in terms of the observable quantities and self-contained. We can obtain not only the low-lying collective excitations but also elucidate their physical origins in a physically transparent manner by linearizing the hydrodynamic equations.