論文の内容の要旨

論文題目：Quantum-theoretical properties of lattice field theories with exact chiral symmetry

（厳密なカイラル対称性を持つ格子場理論の量子論的性質）

氏名：臼井 耕太

In lattice regularization of relativistic quantum field theories, which plays a central role in the analyses of the theories on a non-perturbative level, Euclidean formulation makes it nontrivial whether or not the lattice scheme indeed defines a quantum theory which permits the probability interpretation. Because taking the continuum limit is too difficult a mathematical problem, it is more desirable that a quantum theory can already be reconstructed before taking the continuum limit. Therefore, it is an important issue to study the possibility of reconstructing quantum theories from lattice field theories, and the reflection positivity condition is known to ensure that a Hilbert space of state vectors and a Hamiltonian operator are reconstructed from a lattice model.

Besides the reconstruction problem, it was considered to be impossible to directly deal with the exact chiral symmetry on the lattice, since Nielsen-Ninomiya's no-go theorem was proved. It was the discovery of the overlap Dirac operator which opened up a new way to treat the exact chiral symmetry on the lattice based on the Ginsparg-Wilson relation.

The Ginsparg-Wilson relation expresses the chiral symmetry on the lattice, but other fundamental properties of lattice models, such as locality and possibility of reconstructing a quantum theory, have to be additionally established. The overlap Dirac operator was proved to be local in the sense that it has exponentially decaying tails, but the possibility of reconstructing a corresponding quantum theory was not completely understood.

In this thesis, we will prove that the overlap Dirac fermion satisfies the reflection positivity condition and therefore if the infinite volume limit is assumed the lattice overlap fermion model indeed defines a quantum mechanical system with a Hilbert space of state vectors and self-adjoint energy-momentum operators in it. Furthermore, it will be found that the joint spectrum of energy-momentum operators is

supported on $[0,\infty)\times[-\pi,\pi]^{d-1}$, where d is space-time dimension. This result shows that the Ginsparg-Wilson relation, based on which the exact lattice chiral symmetry is formulated, is compatible with these quantum theoretical properties. It will be also proved that the overlap boson model violates the reflection positivity condition. This implies that the lattice Wess-Zumino model formulated by using overlap boson violates the reflection positivity condition, and one can not be indifferent toward the violation of quantum theoretical unitarity condition when one uses this formulation of the lattice Wess-Zumino model in practical applications. These are main conclusions of this thesis.

In chapter 1, the motivation of studies in this thesis will be presented.

In chapter 2, a mathematical scheme of lattice field theories which contain scalar, spinor, and gauge fields will be given, and Wilson's lattice QCD will be introduced.

In chapter 3, generic lattice models satisfying the basic assumptions **(A1)**-**(A4)** will be considered and it will be proved that these models are ensured to indeed define physically acceptable quantum theoretical systems with strongly commuting Hamiltonian and momentum operators $(H, \boldsymbol{P})$ supported on $[0,\infty)\times[-\pi,\pi]^{d-1}$. It will be also proved that such theories permit the Umezawa-Kamefuchi-Källén-Lehmann spectral representation of the two point correlation functions, with positive spectral density functions. This tells us that positivity of the spectral density is necessary for a model to satisfy the conditions **(A1)**-**(A4)**.

In chapter 4, the overlap Dirac operator will be introduced and basic properties will be proved. It will be shown that, if gauge fields are admissible, then the overlap Dirac operator has exponentially decaying tails.

In chapter 5, the main chapter of this thesis, it will be proved that the overlap Dirac fermion satisfies the reflection positivity condition, and that the overlap boson violates the reflection positivity condition. As an application, the lattice Wess-Zumino model will be discussed in the viewpoint of the refection positivity. It will be pointed out that the gauge interacting case is still an open problem. But the method we will present here will not seem to be used straightforwardly in the gauge interacting case. This will need some new idea.

In chapter 6, we will summarize the thesis.