論文内容の要旨

Static and Dynamic Critical Exponents from Two-Particle-Irreducible $1/N$ Expansion
（二粒子既約 $1/N$ 展開による静的及び動的臨界指数の研究）

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Study of non-equilibrium dynamics become more and more important in many areas of physics such as heavy ion collisions, cold atoms, early universe and so on. Also from theoretical point of view, formalism of non-equilibrium steady state has not been established yet. Non-equilibrium dynamics appears, for example in heavy ion collision, in perception of glasma and to fix a QCD critical end point. Study of QCD at finite temperature and chemical potential has revealed nontrivial phase structure which contains rich physics. When we study phase structure, the critical point plays a very important role. It is well known that when the system in local equilibrium approaches to a critical point, relaxation to equilibrium state takes infinitely long time. This situation is quite different from physics in heavy ion collision, in which relaxation becomes much faster than we expected from perturbation. However, we still consider that understanding of dynamics near critical points is important.

When we theoretically deal with critical phenomena, scaling hypothesis plays a central role. This idea is as follows: Since the correlation length and the relaxation time of the system diverge, the system looks almost the same under scale transformation of the system. Thus physics in a critical region do not depend on details of systems. This is a basic idea of universality class, which was found in experiment.

In critical dynamics, macroscopic effective theory, so called *mode coupling theory*, has often been used to describe critical dynamics and achieved successful results. However, the dynamic universality class of QCD seems still controversial, when mode coupling theory is applied to the critical end point of QCD phase diagram. Also, we desire to understand critical dynamics of QCD from the dynamics of quarks and gluons. Thus, taking a simple field theory, we attempt to deal with dynamic critical phenomena by microscopically. Unfortunately it is almost desperate to describe dynamics in local equilibrium regions by microscopic perturbation theory, even away from equilibrium. The reason is that ordinary perturbations are expansions about number of collisions, but local equilibrium is realized by collisions of infinitely many times. Therefore some resummation is also needed to describe local equilibrium and to obtain transport coefficients. This is why we employ 2PI formalism. 2PI formalism resums higher order diagrams in order that conservation laws are automatically satisfied and leads to hydrodynamic equations, and it helpful to derive transport coefficients.

The goal of this thesis is two-fold. One is to evaluate an off-critical exponent ν in the 2PI formalism. The other is to describe critical dynamics from microscopic theory. We employ 2PI formalism, since it systematically resums higher order contributions of 1PI formalism and automatically satisfies hydrodynamic equations in approximation level. The former property improves a value of ν, and the latter property will be essential for critical dynamics. We take $O(N)$ φ^4 model, which corresponds to, for example, Ising model ($N = 1$), ^{4}He superfluid transition ($N = 2$) and QCD chiral phase transition ($N = 4$). We employ $1/N$ expansion in NLO of φ^4 model.

In static critical phenomena, we can explicitly see diagrams which are resummed in 1PI and 2PI formalisms and confirm that 2PI takes higher order terms in 1PI diagrams. From the scaling relations, if we obtain two exponents (except for two exponents *at* a critical point, which are related to each others), we can completely determine all exponents of the system. Therefore, evaluations of off-critical exponent ν is important. It is also known that 2PI formalism improves the evaluation of η in $1/N$ NLO comparing the 1PI result. Therefore we apply 2PI formalism to improve an evaluation of another (and from scaling relation, the other) exponent ν. In this calculation, we derive a self-consistent equation of a three point vertex function $\Gamma^{(2,1)}(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{z}) \sim \langle \varphi(\boldsymbol{x})\varphi(\boldsymbol{y})\varphi^2(\boldsymbol{z}) \rangle$ (we show the strict definition of $\Gamma^{(2,1)}(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{z})$ in Sec.3), which is related to ν. This relation between $\Gamma^{(2,1)}$ and ν is shown by Ma. Using $\Gamma^{(2,1)}$, we can estimate ν *at* a critical point, although ν is an exponent *near* a critical point.

In dynamic critical phenomena, we attempt to extract dissipative modes, which control dynamics in a critical region and their dispersions are $\omega \sim iDp^2$ (D is a transport coefficient), by calculating a two-point correlation function. Then we evaluate a dynamic critical exponent $\hat{z}$. In 1PI formalism, we fail to extract dissipative modes and there appears propagating modes, whose dispersions are $\omega \sim \pm c_s p + i\Gamma$ (c_s is a sound velocity and Γ is a decay rate). This stems from perturbation that expands correlation functions around a free theory. In contrast, we expect that a correlation function from 2PI effective action has dissipative modes, since they satisfy hydrodynamic equations in approximation level. Hence, we assume that a two-point correlation function in critical region has a dissipative pole,

$$G(\boldsymbol{p}, z) \sim \frac{1}{iD^{-1}z - p^2}. \tag{1}$$

Solving a self-consistent equation, we will obtain $\hat{z}$ from microscopic theory. However, this calculation is solely tentative and we have not evaluated $\hat{z}$ correctly yet.

This thesis is organized as follows. In Sec.2 we show the role of resummation which resolves a secularity problem. And we introduce 2PI effective action comparing ordinary 1PI effective action. Then we show the $1/N$ NLO diagrams in 2PI which we use through this thesis. In Sec.3, we evaluate static critical exponents η and ν, which corresponds to a two-point correlation function *on* and *near* a critical point. We review the evaluation of η in $1/N$ expansion by Ma (1PI) and by Alford, Berges and Cheyne (2PI), then we compare two results and confirm the validity of resummation. Next, we show 1PI estimation of ν (Ma) and calculate by 2PI formalism, then we compare the 1PI and 2PI results. We find that the differences come from diagrams of higher order by comparing resummed diagrams in both 1PI and 2PI formalism. In Sec.4, we proceed to dynamic critical phenomena and try to estimate a dynamic critical exponent $\hat{z}$ from microscopic Hamiltonian.

From linear response theory, we can deal with critical dynamics by correlation functions of equilibrium state. Firstly, we review the conventional estimation of $\hat{z}$; the *mode coupling theory*, and we introduce an action from which we can derive Langevin equations and show perturbation of mode coupling theory. Secondly, we attempt to evaluate a dynamic critical exponent from 1PI effective action in microscopic dynamics. Thirdly, we propose a tentative idea to apply 2PI formalism to critical dynamics. Finally, we discuss evaluations obtained in this section. In Sec.5, we summarize this thesis. Details of some complicated calculations are shown in Appendices.