論文内容の要旨

Correspondence between one dimensional quantum many-body systems and the bosonized effective field theory

(1次元量子多体系とボソン化有効場理論の対応)

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In one dimension, the systems exhibits a unique property due to strong quantum fluctuation. A typical example is that conduction electrons confined in one dimension become Tomonaga-Luttinger (TL) liquid instead of Fermi liquid. Models for strongly correlated electron systems such as the Hubbard and $t-J$ model have been intensively studied in the context of high-temperature superconductors. Recent advancement of experimental techniques realizes one dimensional systems in many kinds of actual materials, e.g. artificial quantum wire manipulated by lithography technology, carbon nanotube, edge states of quantum Hall effect, atomic gas trapped in the optical lattice and quantum spin chains. Another distinctive characteristic of one-dimensional systems is the existence of powerful methods in both analytical and numerical approaches. Critical systems are classified analytically by using conformal field theory, and some models can be solved exactly through Bethe ansatz integral equation. Mapping the models to effective field theory by bosonization is also a useful technique to study low energy excitation structure and correlation functions. On the numerical side, density matrix renormalization group (DMRG), which enables calculations for quite large size system with good accuracy, is a popular method. Other DMRG-like numerical methods including multi-scale entanglement renormalization ansatz, projected entangled-pair states and infinite time-evolving block decimation (iTEBD) have recently been invented and improved. Of course, Quantum Monte Carlo and exact diagonalization (ED) are also available. ED is especially helpful when combined with conformal field theory knowledge as finite size scaling or level spectroscopy.

In this PhD thesis, we address three topics concerning one-dimensional quantum systems.

Two of them are about spin chains and the other is ultracold atomic gas. We approach these systems from analytical and numerical sides. The analytical method is field theoretical one with bosonization and conformal field theory. The employed computational techniques are ED and iTEBD. In iTEBD, the matrix product state of the system evolves in imaginary time direction through Trotter-Suzuki decomposition, leading to the ground state. A merit of this method is that an infinite-size system can be treated directly by imposing the translational symmetry on the system. The detailed principle and implementation is explained in the appendix. We briefly summarize each content of three topics in the following.

(1) Determination of bosonized coefficients in dimer operators

The correspondence between spin-$\frac{1}{2}$ XXZ chain and its effective field theory has been investigated intensively. For example, while spinon velocity and TL parameter are obtained exactly by utilizing Bethe ansatz, bosonization coefficients of spin operators are non-universal and cannot be derived analytically in magnetic field. These coefficients have been determined from the comparison between asymptotic form of correlation function and the DMRG result. We propose a simple new method of deriving the information on uniform spin-$\frac{1}{2}$ XXZ chains from deformed chains. Namely, the coefficients of bosonized dimer operators in spin-$\frac{1}{2}$ XXZ chains are extracted by comparing the numerically calculated gap in dimerized spin chains with the gap formula of the effective sine-Gordon theory. Figure 1 shows that the numerical data are quantitatively fitted by the gap formula of sine-Gordon theory, which supports the validity of our strategy for fixing the coefficients.

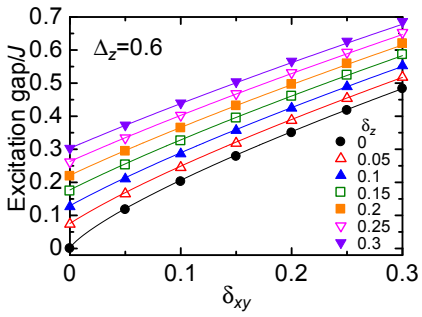

Fig. 1 Numerically evaluated excitation gap of dimerized XXZ chains with exchange anisotropy $\Delta_z = 0.6$ and several values of dimerization in XY-plane δ_{xy} and along Z-axis δ_z. Solid curves are fitting by sine-Gordon theory.

We apply obtained coefficients to the determination of ground-state phase diagrams of dimerized spin chains in a magnetic field and antiferromagnetic frustrated spin ladders with a four-spin interaction. The optical response in one-dimensional Mott insulators with Peierls instability is also evaluated quantitatively.

(2) Mass ratio of excitation particle in dimerized chain with frustration

We study Heisenberg antiferromagnetic (HAF) chain with alternating exchange interaction:

$$\mathcal{H} = \sum_j \left[\{1 + (-1)^j \delta\} J \boldsymbol{S}_j \cdot \boldsymbol{S}_{j+1} + J_2 \boldsymbol{S}_j \cdot \boldsymbol{S}_{j+2}\right]$$

in association with the content of (1). As already mentioned, it is known that this lattice model is related with effective field theory through the bosonization. According to its prediction, there are three kinds of excitation particles, a soliton, anti-soliton and breathers (bound state of soliton and anti-soliton). The lightest breather has the same mass as a soliton and anti-soliton, forming a triplet, and the mass of second lightest breather is $\sqrt{3}$ times as large as that of the triplet. However, numerical calculations for $S = \frac{1}{2}$ indicate that the mass ratio r of the second lightest breather to the triplet deviates from $\sqrt{3}$ unless $J_2/J \sim 0.25$, where the transition from TL liquid to dimerized phase happens. It indicates that r is affected by the marginal term added to sine-Gordon theory, but the quantitative explanation has not been given.

We first demonstrate that the $S = \frac{1}{2}$ and 1 cases are understood in a unified way by using sine-Gordon theory with the marginal term. For both cases, r becomes $\sqrt{3}$ by the introduction of next-nearest neighbor coupling $J_2 = J_{2c}$, where the marginal term in effective field theory vanishes. The universality class of transition is TL liquid and first order for $J_2 < J_{2c}$ and $J_2 > J_{2c}$, respectively. Then, the effect of the marginal term on r quantitatively is considered. We derive the formula for r as a function of δ and J_2/J, combining the result of form factor perturbation theory and renormalization analysis. The result is shown in Fig. 2. The numerically calculated r is well fitted by the derived function with a single parameter.

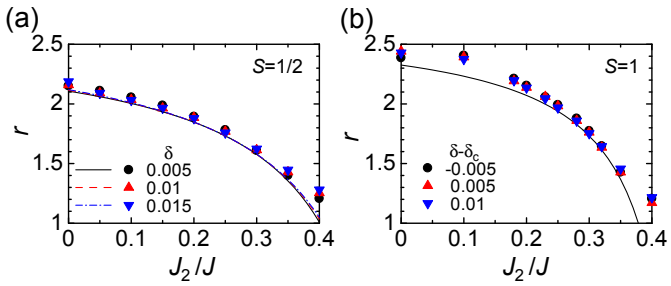

Fig. 2 (a) The case of $S = \frac{1}{2}$. The circle, triangle and down-pointing triangle represent numerically obtained r for $\delta = 0.005$, 0.01 and 0.015, respectively. The solid, dashed and dashed-dotted lines are the derived formula for r as a function of δ and J_2/J. (b) The case of $S = 1$. The circle, triangle and down-pointing triangle represent numerically obtained r for $\delta - \delta_c = -0.005$, 0.005 and 0.01, respectively. Here, δ_c is the transition point of δ from Haldane to dimerized phase. The solid line is the derived formula.

(3) Population imbalance in two-component atomic gas in one dimension

We study two-component (or pseudo spin-$\frac{1}{2}$) Bose or Fermi gases confined in one dimen-

sion, in which particles are convertible between the components. The usual weak-coupling theory based on bosonization predicts that a strong inter-component repulsion induces spontaneous population imbalance between the components, in other words, the ferromagnetism of the pseudo-spins (see Fig. 3). However, we cannot examine any property of ferromagnetic phase and the phase transition from the weak-coupling approach since it breaks down at the transition point. To study the strong-coupling regime, we employ numerical methods (ED and iTEBD). Then it is demonstrated that the imbalanced phase contains gapped spin excitations and gapless charge excitations characterized as TL liquid.

We also uncover a crucial effect of the inter-component particle hopping on the transition to the imbalanced phase. In the absence of this hopping, the transition is of first order. With an infinitesimal inter-component hopping, the transition becomes of Ising type. These results can be qualitatively understood from a simple perturbation theory in the strong-coupling limit. From the accurate numerical data, we determine the ground-state phase diagram in a wide parameter regime and test the reliability of the weak-coupling bosonization formalism.

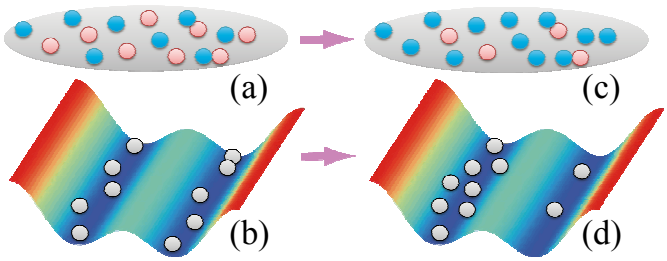

Fig. 3 Illustrations of 1D two-component gases. The two components represent two internal states of atoms [shown by different colors in (a) and (c)]. Even without internal states, a double-channel trap potential can produce a similar situation [(b) and (d)]. A strong repulsion between the components induces the population imbalance [(c) and (d)].