論文の内容の要旨

論文題目　　Degeneracy-Aware Interpolation of Diffusion Tensor Fields and Its Applications

（退化を考慮した拡散テンソル場の補間と応用）

氏　　名　　畢　重科

1 Introduction

Visualizing diffusion tensor fields has become an important topic in many applications. However, it is still difficult to track the underlying continuous behaviors embedded in discrete diffusion tensor fields by employing existing schemes, especially around degenerate points that lead to rotational inconsistency of tensor anisotropy.

We describe our first contribution to smoothly track the continuous behaviors in diffusion tensor fields in Chapter 3 and Chapter 4 (Figure 1). This is accomplished by locating the possible degenerate points globally using a minimum spanning tree (MST) based algorithm firstly. Then we limit the size of isotropic region to avoid that these isotropic tensor samples degrade the anisotropic features of the underlying continuous behaviors in the discrete diffusion tensor fields.

The idea to locate the possible degenerate points globally in the diffusion tensor fields inspires our next contribution demonstrated in Chapter 5 (Figure 1) to control the mesh topology in quadrilateral mesh through introducing a 2D diffusion tensor field. This is because the streamlines along the two principal directions of a tensor field and a quadrilateral mesh are dual to each other. The region containing a degenerate point that is located by using our MST algorithm is dual to an extraordinary (i.e. non-degree-four) vertex in a quadrilateral mesh,

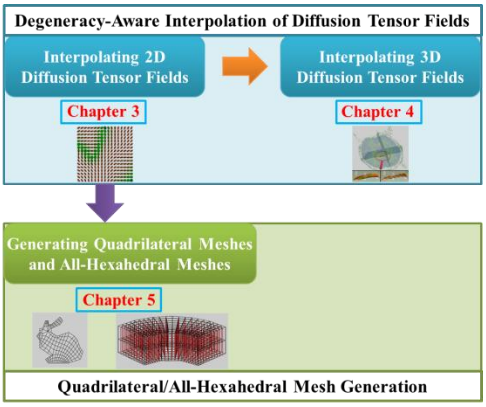

Figure 1: Organization of this thesis

while the non-degenerate region is dual to an ordinary vertex in the quadrilateral mesh. Furthermore, all-hexahedral mesh is also generated by using sweeping operations.

2 Degeneracy-Aware Interpolation of 2D Diffusion Tensor Fields

Our primary idea [1] for interpolating 2D diffusion tensor fields is to locate and resolve tensor degeneracy for tracking smooth transitions of anisotropic features inherent in the given data.

For locating the possible tensor degeneracy, we employ a minimum spanning tree (MST) strategy to group discrete tensor samples with similar orientations of the anisotropic features. For this purpose, we define the similarity between a pair of neighboring tensor samples as:

$$\begin{aligned} d(D^S, D^T) &= \alpha\left|C_l^S - C_l^T\right| \\ &+ \beta\left|C_p^S - C_p^T\right| + \gamma(\left|\theta^{S,T}\right|/(\pi/2)) \end{aligned}, \qquad (1)$$

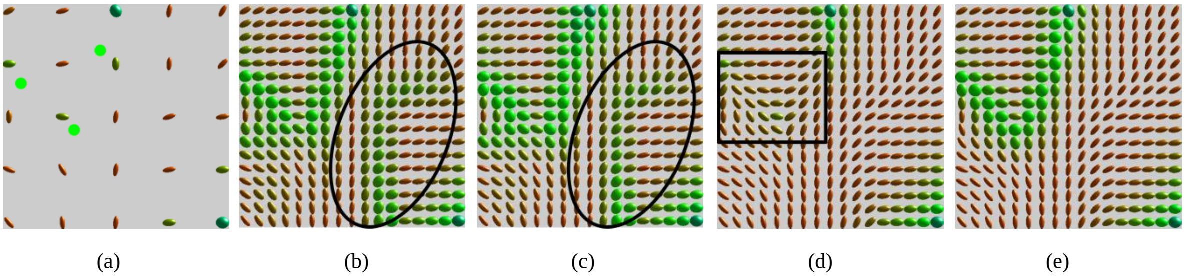

Figure 2: Interpolating a diffusion tensor field containing three degenerate points (represented by green circles). (a) Original tensor samples. The results with the (b) component-wise and (c) log-Euclidean. These interpolation methods cannot retain the anisotropic feature inherent in the original tensor field, as indicated by the black circle. (d) The result with geodesic-loxodrome. This usually incurs the problem of rotation inconsistency around degenerate points, as shown by the black rectangle. (e) The result obtained by our interpolation scheme. Our scheme can fully respect the high anisotropic features, and take care of the rotational inconsistency around degenerate points.

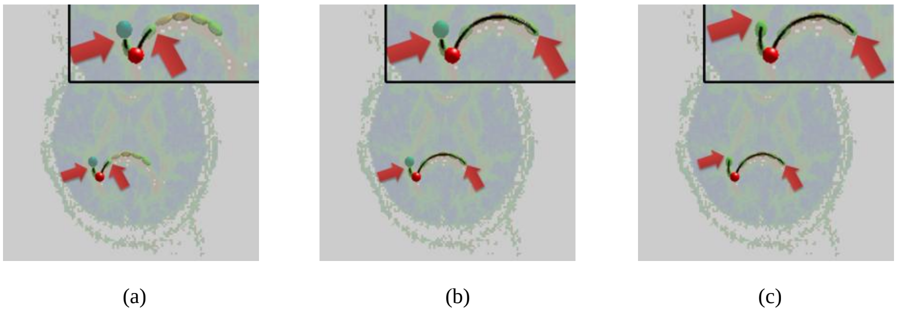

Figure 3: Tracking two fibers in a human brain DT-MRI dataset, where the red point is the seed point. Several degenerate points exist between the two fibers. Interpolated results with the (a) component-wise and (b) log-Euclidean cannot fully track the two fibers due to the degeneracy in the region between the two fibers, while our scheme can successfully track the two fibers and avoid the influence of existing degeneracy. This is because we limit the size of the isotropic region while maximally respecting the anisotropy of the fibers.

where $\boldsymbol{C}_l^S$ and $\boldsymbol{C}_l^S$ represent the $\boldsymbol{C}_l$ values of the two tensors $\boldsymbol{D}^S$ and $\boldsymbol{D}^T$, and $\boldsymbol{C}_p^S$ and $\boldsymbol{C}_p^T$ represent the corresponding $\boldsymbol{C}_p$ values. In addition, $\boldsymbol{\theta}^{S,T}$ is the minimal rotation angle between the right-handed coordinate systems defined by the two sets of eigenvector directions. Here, we locate degenerate points by counting the number of *degenerate pairs* in one cell, where a degenerate pair is defined as a pair of tensors whose rotational angle is larger than $\pi/2$.

The rotational inconsistency is resolved by optimizing the rotational transformation between a pair of neighboring tensors through analyzing their associated eigenstructure.

Figure 2 shows a result where a 2D tensor field with three degenerate points is interpolated.

3 Degeneracy-Aware Interpolation of 3D Diffusion Tensor Fields

In 3D diffusion tensor fields, it becomes much more difficult to locate and resolve tensor degeneracy since the topological structure of degeneracy is much more complicated.

When constructing a minimum spanning tree in 3D space, we often encounter unwanted cases where an important pair of tensor samples are left unconnected, especially when the rotation angle between the primary eigenvectors at the two end tensor samples becomes close to 0 while those between other pairs of eigenvectors approach to

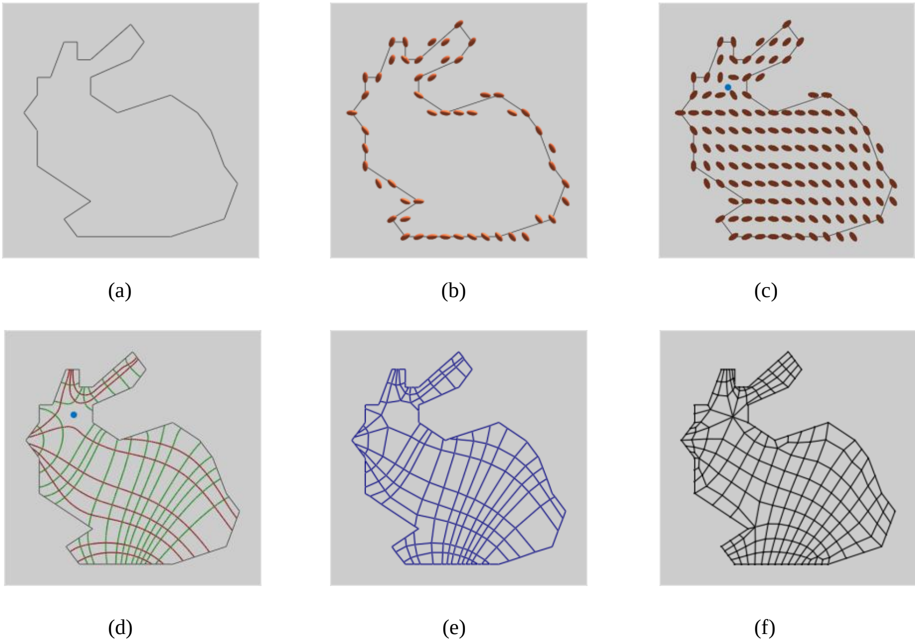

Figure 4: Generating a quadrilateral mesh in the interior of the target object. (a) 2D bunny shape. (b) The diffusion tensor field along the boundary of the bunny object. (c) The generated diffusion tensor field in the interior of the bunny object. The blue point represents a detected degenerate point. (d) The streamlines of the diffusion tensor fields. (e) The dual graph of the quadrilateral mesh obtained from the streamlines in (d). (f) The quadrilateral mesh obtained from its dual graph in (e).

$\pi/2$. For avoiding this, we revise the previous similarity metric as the following new one [2]:

$$\begin{aligned} d(D^S, D^T) &= \alpha\left|C_l^S - C_l^T\right| + \beta\left|C_p^S - C_p^T\right| \\ &+ \gamma(\left|\theta_1^{S,T}/(\pi/2)\right|) + \delta(\left|\theta_2^{S,T}/(\pi/2)\right|), \\ &+ (\left|\lambda_1^S - \lambda_1^T\right|/(\lambda_1^{MAX} - \lambda_1^{MIN})) \end{aligned} \tag{2}$$

where the fourth term is newly introduced to evaluate the minimum rotational angle between the two primary eigenvectors since they are the most important for fiber tracking. The fifth term is employed to discriminate between the two tensors with the same anisotropy orientation but different size.

For resolving rotational inconsistency, we minimize an objective function so as to transform each degenerate pair to non-degenerate one as well as to minimize the number of newly generated degenerate pairs.

Figure 3 shows a result where the two fibers that run around the tensor degeneracy have been tracked in a human brain dataset.

4 Quadrilateral/Hexahedral Mesh Generation based on Tensor Fields

We proposed an approach [3] to interactively controlling the mesh topology of quadrilateral meshes by introducing diffusion tensor fields into the target object.

Firstly, the Poisson equation is employed to generate the tensor field (Fig. 4(c)) by propagating the tensor anisotropic features along the boundary (Fig. 4(b)) into the interior of the target object (Fig. 4(a)). The possible degenerate points are also located by using our MST-based algorithm.

Then, the streamlines of the diffusion tensor field (Fig. 4(d)) can then be transformed into the dual graph (Fig. 4(e)) of the quadrilateral mesh (Fig. 4(f)), since the streamlines and quadrilateral mesh are dual to each other. Furthermore, our approach allows us to interactively control the mesh topology by changing the orientations of the tensor samples on the boundary of the target object.

Finally, we extend the framework of quadrilateral mesh to generate all-hexahedral mesh using sweeping method.

5 Conclusions and Future Work

In this thesis, we proposed a degeneracy-aware interpolation approach for diffusion tensor fields, which can successfully allow us to track the underlying anisotropic features such as nerve and muscle fibers. This has been achieved by using an MST-based algorithm to locate the possible degenerate points, and resolving such degeneracy by optimizing the rotation transformation between each pair of tensors.

We also introduced an approach to interactively controlling the mesh topology of quadrilateral meshes by introducing a 2D diffusion tensor field into a target object. Finally, all-hexahedral mesh is also generated when the target object can be composed through sweeping operations.

Our future extension includes the challenge to extend our framework to generate all-hexahedral meshes in the volume without constructed structure.

References

[1] C. Bi, S. Takahashi, and I. Fujishiro, "Interpolation 3D Diffusion Tensors in 2D Planar Domain by Locating Degenerate Lines," in *Proceedings of the 6th International Symposium on Visual Computing (ISVC2010)*, Springer LNCS Vol. 6543, pp. 328-337, November, 2010.

[2] C. Bi, S. Takahashi, and I. Fujishiro, "Degeneracy-Aware Interpolation of 3D Diffusion Tensor Fields", in *Proceedings of SPIE Visualization and Data Analysis 2012*, accepted, 2012.

[3] C. Bi, D. Sakurai, and S. Takahashi, "Interactive Control of Mesh Topology in Quadrilateral Mesh Generation based on 2D Tensor Fields", Submitted to *Geometric Modeling and Processing (GMP2012)*.