論文の内容の要旨

Noise-Assisted Computation with Logical Stochastic Resonance

(論理的確率共振を用いた雑音緩用計算)

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With the continuous shrinking and miniaturization of electronic circuits and the ensuing lower voltages and intensities, random fluctuations or noise, be it unavoidable like thermal and shot noise or due to defects in the semiconductors, is becoming a phenomenon that cannot be ignored and that can effectively create an upper bound to computing speed. Moreover new forms of computing, where noise has an inextricable role like DNA, peptide or general chemical computing, are becoming commonplace. We are thus increasingly encountering fundamental noise characteristics that cannot be suppressed or eliminated. In chemical and DNA computing in particular, because of the nature of the systems considered, noise plays a particularly pervasive role. Not only the computation but also the inputs are characterized by strong fluctuations. And this can be for various reasons, one being low number of molecules in certain particular systems like inside cells.

Luckily, noise can also have a constructive function as shown with the discovery of Stochastic Resonance or SR. This concept originally introduced to explain the synchronization between glaciations, a macroscopic phenomenon, and variations in the earth's eccentricity, of small magnitude and resulting in a 0.1 % of solar energy influx, is defined as the mechanism by which a nonlinear dynamical system is driven (i.e. its dynamics are controlled) by a weak external forcing, through an interplay of noise at the appropriate intensity and nonlinearity. Being possible in almost any physical system that can be described a double well, bistable or excitable system with noise and a weak periodic forcing, Stochastic resonance is a very general and universal phenomenon and was discovered in a variety of natural systems. This universality together with the fact that in SR, noise is actually increasing/enhancing signal, not destroying it, make of it a

very promising and interesting mechanism.

A main focus of research on SR is the relation between the frequency of the weak forcing and the level of noise needed for optimal system response. Noise can, however, also enhance weak signals that are not periodic. In 2009, Prof. Sinha introduced the concept of Logical Stochastic Resonance, or LSR, in which SR is used to combine pairs of logic inputs into specific logic outputs. In the study, Sinha presented a bistable dynamical system in which signals represented by square waves of small amplitude, are enhanced by SR to yield logic behavior. Noise in this system does not degrade computation, rather, noise facilitates it. In simpler words, Sinha introduced dynamical systems that act as logic gates in presence of noise at the right intensity. As with stochastic resonance, LSR works in a range of noise intensities and with weak inputs. As SR it is a very universal mechanism and can thus be found in a variety of physical systems.

The original paper by Sinha provided a proof of concept of the new mechanism. In this thesis I analyze and extend the ideas of LSR. The subject is still very young and thus methods, techniques, ideas, etc. are not established yet. Therefore, instead of diving deep in the analysis of some subproblem, I focused on many different aspects and ways of looking at LSR and decided leave the study of the details for when there will more clarity. This means that my efforts mainly went on extending the vanilla idea of a bistable system with continuous one-dimensional dynamics to the more interesting and multistable, multidimensional and discrete time alternatives. The main results of this thesis are therefore a range of new dynamical systems able to perform LSR style computation.

Chapter 1, which serves as introduction to the thesis, provides an introduction and a review of three very hot very research fields in which noise plays an important and detrimental role: highly miniaturized electronic logic gates, chemical computing and micro- and nanoelectromechanical systems (MEMS and NEMS). After looking at he negative points of noise we explore its positive side with examples of constructive ways to use noise. The first one and maybe more famous are brownian motors, devices that extract energy from noise. In the second part we talk about stochastic resonance as way to enhance weak signals. All this leads then to the introduction of Logical Stochastic Resonance.

In Chapter 2, we dig deep into the field of stochastic resonance, starting with a physical or intuitive explanation and proceeding to a more mathematical/analytical analysis. A set of mathematical tools commonly used in the quantification of stochastic resonant behavior is also provided.

The natural next step, Logical Stochastic Resonance, is introduced in Chapter 3. Here the double well potential system yielding logic gate like behavior presented by Sinha is illustrated and the robustness of its fundamental parameters is analyzed.

In Chapter 4 we present an extension of Sinha's double well model: the multistable triple well system. Here it is shown that the increase of number of wells, although appearing more a cosmetic change on the surface, has a profound impact on the logic behavior capabilities and versatility of the systems. In practice, this translates into having a simple one dimensional system with the capability to act as XOR logic gate, which was not possible with the two wells system.

In Chapter 5, Logical Stochastic Resonance is extended to multidimensional systems with the adaptation to logic gate of the synthetic gene model known as toggle switch. This model reveals to be very versatile and in addition to having the capability of various logic behavior, its additional variable allows to parallelly compute the results of complementary logic gates. Based on this a more general, polynomial based two dimensional system is proposed.

In Chapter 6, the focus is on another interesting modeling tool: iterated maps. We show how stochastic resonance and logical stochastic resonance are possible in discrete time systems and provide three different maps with various logical behaviors.

In Chapter 7, we analyze LSR gates in the context of digital LSR circuits, that is chained gates. Here, we first analyze the sigmoidal coupling method and then discuss performance quantifiers to analyze the coupling efficacy.