We propose hyper-lambda calculi, the typed lambda calculi based on hypersequent calculi. A hyper-lambda term is a finite sequence of lambda terms, representing concurrent processes. We give two concrete hyper-lambda calculi: synchronous and asynchronous. Both employ a pair of communication primitives exchanging their inputs. In the synchronous case, both sides succeed. In the asynchronous case, at least one side obtains the other side’s input. The synchronous calculus implements message-passing communication and session types; the asynchronous calculus characterizes shared-memory waitfree communication. Among processes of a typed hyper-lambda term, all succeed in the synchronous case while at least one succeeds in the asynchronous case. Logically, the processes are interpreted conjunctively in the synchronous case but disjunctively in the asynchronous case. The synchronous calculus is based on Abelian logic: $(\varphi \to \psi) \otimes (\psi \to \varphi)$ on top of multiplicative additive fragment of intuitionistic linear logic (without some units); the asynchronous calculus is based on Gödel-Dummett logic: $(\varphi \supset \psi) \lor (\psi \supset \varphi)$ on top of intuitionistic logic. The hyper-lambda calculi are in Curry–Howard correspondence with the deduction systems for these logics. We also treat another variant based on monoidal t-norm logic and an implementation using Haskell.

Meanwhile, we discover a new representation of Abelian logic proofs called Amida nets. Amida nets are based on a special kind of directed graphs called Lamarche’s essential nets. On top of Lamarche’s essential nets, we add a new kind of undirected edges called Amida edges. Similarly to the Amida lotteries used by Japanese children, when we follow a directed path in an Amida net, whenever we meet an Amida edge, we have to cross the Amida edge. While Lamarche’s essential nets characterize fragments of intuitionistic linear logic, our Amida nets characterize Abelian logic. The name Amida calculus comes from the Amida lotteries.

The logics treated in this thesis, Gödel–Dummett, monoidal t-norm and Abelian logics were initially developed for algebraic interests, but we reveal that these logics have computational applications as type systems for concurrent processes.